Honors Physics Review Notes

2008–2009

Tom Strong
Science Department
Mt Lebanon High School
tomstrong@gmail.com

The most recent version of this can be found at http://www.tomstrong.org/physics/

These notes are meant to be a summary of important points covered in the Honors Physics class at Mt. Lebanon High School. They are not meant to be a replacement for your own notes that you take in class, nor are they a replacement for your textbook. Much of the material in here is taken from the textbook without specifically acknowledging each case, in particular the organization and overall structure exactly match the 2002 edition of *Holt Physics* by Serway and Faughn and many of the expressions of the ideas come from there as well.

The mixed review exercises were taken from the supplementary materials provided with the textbook. They are a representative sampling of the type of mathematical problems you may see on the final exam for the course. There will also be conceptual questions on the exam that may not be covered by the exercises included here. These exercises are provided to help you to review material that has not been seen in some time, they are not meant to be your only resource for studying. Exercises are included from chapters 1–8, 12–17, and 19–22.

The answers at the end of the review are taken from the textbook, often without verifying that they are correct. Use them to help you to solve the problems but do not accept them as correct without verifying them yourself.

This is a work in progress and will be changing and expanding over time. I have attempted to verify the correctness of the information presented here, but the final responsibility there is yours. Before relying on the information in these notes please verify it against other sources.

Chapter 1 — The Science of Physics

1.1 What is Physics?

Some major areas of Physics:

Mechanics — motion and its causes — falling objects, friction, weight

Thermodynamics — heat and temperature — melting and freezing processes, engines, refrigerators

Vibrations and Waves — specific types of repeating motions — springs, pendulums, sound

Optics — light — mirrors, lenses, color

Electromagnetism — electricity, magnetism, and light — electrical charge, circuitry, magnets

Relativity — particles moving at very high speeds — particle accelerators, particle collisions, nuclear energy

Quantum Mechanics — behavior of sub-microscopic particles — the atom and its parts

The steps of the Scientific Method

1. Make observations and collect data that lead to a question
2. Formulate and objectively test hypotheses by experiments (sometimes listed as 2 steps)
3. Interpret results and revise the hypotheses if necessary
4. State conclusions in a form that can be evaluated by others

1.2 Measurements in Experiments

Measurements

There are 7 basic dimensions in SI (Système International), the 3 we will use most often are:

- Length — **meter** (m) — was 1/10,000,000 of the distance from the equator to the North Pole — now the distance traveled by light in 3.3×10^{-9} s
- Mass — **kilogram** (kg) — was the mass of 0.001 cubic meters of water, now the mass of a specific platinum-iridium cylinder
- Time — **second** (s) — was a fraction of a mean solar day, now 9,162,631,700 times the period of a radio wave emitted by a Cesium-133 atom

Common SI Prefixes

Prefix	Multiple	Abbrev.			
nano-	10^{-9}	n	deci-	10^{-1}	d
micro-	10^{-6}	μ	kilo-	10^{3}	k
milli-	10^{-3}	m	mega-	10^{6}	M
centi-	10^{-2}	c	giga-	10^{9}	G

Accuracy vs. Precision

- **Accuracy** describes how close a measured value is to the true value of the quantity being measured

  Problems with accuracy are due to error. To avoid error:

  - Take repeated measurements to be certain that they are consistent (avoid human error)
  - Take each measurement in the same way (avoid method error)
  - Be sure to use measuring equipment in good working order (avoid instrument error)

- **Precision** refers to the degree of exactness with which a measurement is made and stated.

  - 1.325 m is more precise than 1.3 m
  - lack of precision is usually a result of the limitations of the measuring instrument, not human error or lack of calibration
  - You can estimate where divisions would fall between the marked divisions to increase the precision of the measurement

1.3 The Language of Physics

There are many symbols that will be used in this class, some of the more common will be:

Symbol	Meaning
Δx	Change in x
x_i, x_f	Initial, final values of x
$\sum F$	Sum of all F

Dimensional analysis provides a way of checking to see if an equation has been set up correctly. If the units resulting from the calculation are not those that are expected then it's very unlikely that the numbers will be correct either. **Order of magnitude estimates** provide a quick way to evaluate the appropriateness of an answer — if the estimate doesn't match the answer then there's an error somewhere.

Counting Significant Figures in a Number

Rule	Example
All counted numbers have an infinite number of significant figures	10 items, 3 measurements
All mathematical constants have an infinite number of significant figures	$1/2$, π, e
All nonzero digits are significant	42 has two significant figures; 5.236 has four
Always count zeros between nonzero digits	20.08 has four significant figures; 0.00100409 has six
Never count leading zeros	042 and 0.042 both have two significant figures
Only count trailing zeros if the number contains a decimal point	4200 and 420000 both have two significant figures; 420. has three; 420.00 has five
For numbers in scientific notation apply the above rules to the mantissa (ignore the exponent)	4.2010×10^{28} has five significant figures

Counting Significant Figures in a Calculation

Rule	Example
When adding or subtracting numbers, find the number which is known to the fewest decimal places, then round the result to that decimal place.	$21.398 + 405 - 2.9 = 423$ (3 significant figures, rounded to the ones position)
When multiplying or dividing numbers, find the number with the fewest significant figures, then round the result to that many significant figures.	$0.049623 \times 32.0/478.8 = 0.00332$ (3 significant figures)
When raising a number to some power count the number's significant figures, then round the result to that many significant figures.	$5.8^2 = 34$ (2 significant figures)
Mathematical constants do not influence the precision of any computation.	$2 \times \pi \times 4.00 = 25.1$ (3 significant figures)
In order to avoid introducing errors during multi-step calculations, keep extra significant figures for intermediate results then round properly when you reach the final result.	

Rules for Rounding

Rule	Example
If the hundredths digit is 0 through 4 drop it and all following digits.	1.334 becomes 1.3
If the hundredths digit is 6 though 9 round the tenths digit up to the next higher value.	1.374 becomes 1.4
If the hundredths digit is a 5 followed by other non-zero digits then round the tenths digit up to the next higher value.	1.351 becomes 1.4
If the hundredths digit is a 5 not followed by any non-zero digits then if the tenths digit is even round down, if it is odd then round up.	1.350 becomes 1.4, 1.250 becomes 1.2
(assume that the result was to be rounded to the nearest 0.1, for other precisions adjust accordingly)	

Chapter 2 — Motion in One Dimension

2.1 Displacement and Velocity

The **displacement** of an object is the straight line (vector) drawn from the object's initial position to its new position. Displacement is independent of the path taken and is not necessarily the same as the distance traveled. Mathematically, displacement is:

$$\Delta x = x_f - x_i$$

The **average velocity**, equal to the constant velocity necessary to cover the given displacement in a certain time interval, is the displacement divided by the time interval during which the displacement occurred, measured in $\frac{\mathrm{m}}{\mathrm{s}}$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The **instantaneous velocity** of an object is equivalent to the slope of a tangent line to a graph of x vs. t at the time of interest.

The area under a graph of instantaneous velocity vs. time (v vs. t) is the displacement of the object during that time interval.

2.2 Acceleration

The **average acceleration** of an object is the rate of change of its velocity, measured in $\frac{\mathrm{m}}{\mathrm{s}^2}$. Mathematically, it is:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Like velocity, acceleration has both magnitude and direction. The speed of an object can increase or decrease with either positive or negative acceleration, depending on the direction of the velocity — negative acceleration does not always mean decrease in speed.

The **instantaneous acceleration** of an object is equivalent to the slope of a tangent line to the v vs. t graph at the time of interest, while the area under a graph of instantaneous acceleration vs. time (a vs. t) is the velocity of the object. In this class acceleration will almost always be constant in any problem.

Displacement and velocity with constant uniform acceleration can be expressed mathematically as any of:

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

2.3 Falling Objects

In the absence of air resistance all objects dropped near the surface of a planet fall with effectively the same constant acceleration — called **free fall**. That acceleration is always directed downward, so in the customary frame of reference it is negative, so:

$$a_g = g = -9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Chapter 3 — Two-Dimensional Motion and Vectors

3.1 Introduction to Vectors

Vectors can be added graphically.

Vectors can be added in any order:

$$\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$$

To subtract a vector you add its opposite:

$$\vec{V}_1 - \vec{V}_2 = \vec{V}_1 + (-\vec{V}_2)$$

Multiplying a vector by a scalar results in a vector in same direction as the original vector with a magnitude equal to the original magnitude multiplied by the scalar.

3.2 Vector Operations

To add two perpendicular vectors use the Pythagorean Theorem to find the resultant magnitude and the inverse of the tangent function to find the direction: $V_r = \sqrt{V_x^2 + V_y^2}$ and the direction of V_r, θ_r, is $\tan^{-1} \frac{V_y}{V_x}$

Just as 2 perpendicular vectors can be added, any vector can be broken into two perpendicular component vectors: $\vec{V} = \vec{V}_x + \vec{V}_y$ where $\vec{V}_x = V \cos\theta \hat{\imath}$ and $\vec{V}_y = V \sin\theta \hat{\jmath}$;

Two vectors with the same direction can be added by adding their magnitudes, the resultant vector will have the same direction as the vectors that were added.

Any two (or more) vectors can be added by first decomposing them into component vectors, adding all of the x and y components together, and then adding the two remaining perpendicular vectors as described above.

3.3 Projectile Motion

The equations of motion introduced in chapter 2 are actually vector equations. Once the new symbol for displacement, $\vec{d} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$ has been introduced the most common ones can be rewritten as

$$\vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

Since motion in each direction is independent of motion in the other, objects moving in two dimensions are easier to analyze if each dimension is considered separately.

$$\vec{d}_x = \vec{v}_{xi}\Delta t + \frac{1}{2}\vec{a}_x \Delta t^2$$

$$\vec{d}_y = \vec{v}_{yi}\Delta t + \frac{1}{2}\vec{a}_y \Delta t^2$$

In the specific case of **projectile motion** there is no acceleration in the horizontal ($\hat{\imath}$) direction ($\vec{a}_x = 0$) and the acceleration in the vertical ($\hat{\jmath}$) direction is constant ($\vec{a}_y = \vec{a}_g = \vec{g} = -9.81 \, \frac{\mathrm{m}}{\mathrm{s}^2}\hat{\jmath}$). While $\vec{v}_y$ will change over time, $\vec{v}_x$ remains constant. Neglecting air resistance, the path followed by an object in projectile motion is a parabola. The following equations use these simplifications to describe the motion of a projectile launched with speed $\vec{v}_i$ and direction θ up from horizontal:

$$\vec{v}_x = v_i \cos\theta \hat{\imath} = \text{constant}$$

$$\vec{v}_{yi} = v_i \sin\theta \hat{\jmath}$$

$$\vec{d}_x = v_i \cos\theta \Delta t \hat{\imath}$$

$$\vec{d}_y = v_i \sin\theta \Delta t \hat{\jmath} + \frac{1}{2}\vec{g}\Delta t^2$$

To solve a projectile motion problem use one part of the problem to find Δt, then once you know Δt you can use that to solve the rest of the problem. In almost every case this will involve using the vertical part of the problem to find Δt which will then let you solve the horizontal part but there may be some problems where the opposite approach will be necessary. For the special case of a projectile launched on a level surface the range can be found with

$$d_x = \frac{v^2 \sin 2\theta}{a_g}$$

Chapter 4 — Forces and the Laws of Motion

4.1 Changes in Motion

Force causes change in velocity. It can cause a stationary object to move or a moving object to stop or otherwise change its motion.

The unit of force is the **newton** (N), equivalent to $\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$, which is defined as the amount of force that, when acting on a 1 kg mass, produces an acceleration of $1\frac{\text{m}}{\text{s}^2}$.

Contact forces act between any objects that are in physical contact with each other, while **field forces** act over a distance.

A **force diagram** is a diagram showing all of the forces acting on the objects in a system.

A **free-body diagram** is a diagram showing all of the forces acting on a single object isolated from its surroundings.

4.2 Newton's First Law

Newton's first law states that

> An object at rest remains at rest, and an object in motion continues in motion with constant velocity (that is, constant speed in a straight line) unless the object experiences a net external force.

This tendency of an object to not accelerate is called **inertia**. Another way of stating the first law is that if the net external force on an object is zero, then the acceleration of that object is also zero. Mathematically this is

$$\sum \vec{F} = 0 \rightarrow \vec{a} = 0$$

An object experiencing no net external force is said to be in **equilibrium**, if it is also at rest then it is in **static equilibrium**

4.3 Newton's Second and Third Laws

Newton's second law states that

> The acceleration of an object is directly proportional to the net external force acting on the object and inversely proportional to the object's mass.

Mathematically this can be stated as:

$$\sum \vec{F} = m\vec{a}$$

Which in the case of no net external force ($\sum \vec{F} = 0$) also illustrates the first law:

$$\sum \vec{F} = 0 \rightarrow m\vec{a} = 0 \rightarrow \vec{a} = 0$$

Newton's third law states that

> If two objects interact, the magnitude of the force exerted on object 1 by object 2 is equal to the magnitude of the force simultaneously exerted on object 2 by object 1, and these two forces are opposite in direction.

Mathematically, this can be stated as:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

The two equal but opposite forces form an **action-reaction pair**.

4.4 Everyday Forces

The **weight** of an object ($\vec{F}_g$) is the gravitational force exerted on the object by the Earth. Mathematically:

$$\vec{F}_g = m\vec{g} \text{ where } \vec{g} = -9.81\frac{\text{m}}{\text{s}^2}\hat{\jmath}$$

The **normal force** ($\vec{F}_n$) is the force exerted on an object by the surface upon which the object rests. This force is always perpendicular to the surface at the point of contact.

The force of **static friction** ($\vec{F}_s$) is the force that opposes motion before an object begins to move, it will prevent motion so long as it has a magnitude greater than the applied force in the direction of motion. The maximum magnitude of the force of static friction is the product of the magnitude of the normal force times the **coefficient of static friction** (μ_s) and its direction is opposite the direction of motion:

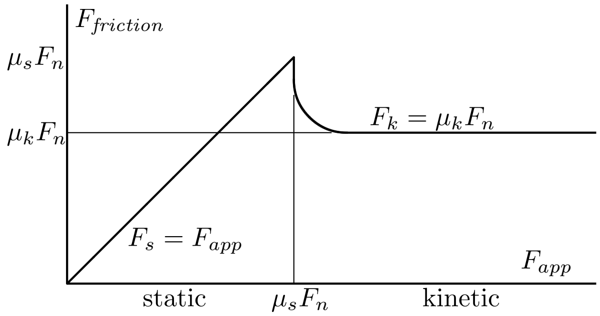
$$\mu_s = \frac{F_{s,max}}{F_n} \rightarrow F_{s,max} = \mu_s F_n$$

The force of **kinetic friction** ($\vec{F}_k$) is the force that opposes the motion of an object that is sliding against a surface. The magnitude of the force of kinetic friction is the product of the magnitude of the normal force times the **coefficient of kinetic friction** (μ_k) and its direction is opposite the direction of motion:

$$\mu_k = \frac{F_k}{F_n} \rightarrow F_k = \mu_k F_n$$

The force of friction between two solid objects depends only on the normal force and the coefficient of friction, it is independent of the the surface area in contact between them.

For an object at rest with a force applied to it the frictional force will vary as the applied force is increased.



Air resistance ($\vec{F}_R$) is the force that opposes the motion of an object though a fluid. For small speeds $F_R \propto v$ while for large speeds $F_R \propto v^2$. (Exactly what is small or large depends on things that are outside the scope of this class.) Air resistance is what will cause falling objects to eventually reach a **terminal speed** where $F_R = F_g$.

Solving Friction Problems

When solving friction problems start by drawing a diagram of the system being sure to include the gravitational force (F_g), normal force (F_n), frictional force (F_k, F_s, or $F_{s,max}$) and any applied force(s).

Determine an appropriate frame of reference, rotating the x and y coordinate axes if that is convenient for the problem (such as an object on an inclined plane). Resolve any vectors not lying along the coordinate axes into components.

Use Newton's second law ($\sum \vec{F} = m\vec{a}$) to find the relationship between the acceleration of the object (often zero) and the applied forces, setting up equations in both the x and y directions ($\sum \vec{F}_x = m\vec{a}_x$ and $\sum \vec{F}_y = m\vec{a}_y$) to solve for any unknown quantities.

Chapter 5 — Work and Energy

5.1 Work

A force that causes a displacement of an object does **work** (W) on the object. The work is equal to the product of the distance that the object is displaced times the component of the force in the direction of the displacement, or if θ is the angle between the displacement vector and the (constant) net applied force vector, then:

$$W = F_{net} d \cos\theta$$

The units of work are **Joules** (J) which are equivalent to $\mathrm{N} \cdot \mathrm{m}$ or $\frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$. The sign of the work being done is significant, it is possible to do a negative amount of work.

5.2 Energy

Work done to change the speed of an object will accumulate as the **kinetic energy** (KE, or sometimes K) of the object. Kinetic energy is:

$$KE = \frac{1}{2} m v^2$$

If work is being done on an object, the **work-kinetic energy theorem** shows that:

$$W_{net} = \Delta KE$$

In addition to kinetic energy, there is also **potential energy** (PE, or sometimes U) which is the energy stored in an object because of its position. If the energy is stored by lifting the object to some height h, then the equation for **gravitational potential energy** is:

$$PE_g = mgh$$

Potential energy can also be stored in a compressed or stretched spring, if x is the distance that a spring is stretched from its rest position and k is the **spring constant** measuring the stiffness of the spring (in newtons per meter) then the **elastic potential energy** that is stored is:

$$PE_{elastic} = \frac{1}{2} k x^2$$

The units for all types of energy are the same as those for work, Joules.

5.3 Conservation of Energy

The sum an objects kinetic energy and potential energy is the object's **mechanical energy** (ME). In the absence of friction, the total mechanical energy of a system will remain the same. Mathematically, this can be expressed as:

$$ME_i = ME_f$$

In the case of a single object in motion, this becomes:

$$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f$$

5.4 Power

The rate at which energy is transferred is called **power** (P). The mathematical expression for power is:

$$P = \frac{W}{\Delta t} = \frac{Fd}{\Delta t} = F \frac{d}{\Delta t} = Fv$$

The units for power are **Watts** (W) which are equivalent to $\frac{\mathrm{J}}{\mathrm{s}}$ or $\frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^3}$

Chapter 6 — Momentum and Collisions

6.1 Momentum and Impulse

Momentum is a vector quantity described by the product of an object's mass times its velocity:

$$\vec{p} = m\vec{v}$$

If an object's momentum is known its kinetic energy can be found as follows:

$$KE = \frac{p^2}{2m}$$

The change in the momentum of an object is equal to the **impulse** delivered to the object. The impulse is equal to the constant net external force acting on the object times the time over which the force acts:

$$\Delta\vec{p} = \vec{F}\Delta t$$

Any force acting on on object will cause an impulse, including frictional and gravitational forces.

In one dimension the slope of a graph of the momentum of an object vs. time is the net external force acting on the object. The area under a graph of the net external force acting on an object vs. time is the total change in momentum of that object.

Momentum and impulse are measured in $\frac{\text{kg}\cdot\text{m}}{\text{s}}$ — there is no special name for that unit.

6.2 Conservation of Momentum

Momemtum is always conserved in any closed system:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

For two objects, this becomes:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Any time two objects interact, the change in momentum of one object is equal in magnitude and opposite in direction to the change in momentum of the other object:

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

6.3 Elastic and Inelastic Collisions

There are three types of collisions:

- **Elastic** — momentum and kinetic energy are conserved. Both objects return to their original shape and move away separately. Generally:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

$$\Sigma KE_i = \Sigma KE_f$$

For two objects:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

- **Inelastic** — momentum is conserved, kinetic energy is lost. One or more of the objects is deformed in the collision. Generally:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

$$\Sigma KE_i > \Sigma KE_f$$

For two objects:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 > \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

- **Perfectly inelastic** — momentum is conserved, kinetic energy is lost. One or more objects may be deformed and the objects stick together after the collision. Generally:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

$$\Sigma KE_i > \Sigma KE_f$$

For two objects:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 > \frac{1}{2}(m_1 + m_2)v_f^2$$

True elastic or perfectly inelastic collisions are very rare in the real world. If we ignore friction and other small energy losses many collisions may be modeled by them.

Newton's Laws in terms of Momentum

1. Inertia:
$$\Sigma\vec{F} = 0 \rightarrow \vec{p} = constant$$
2. $\vec{F} = m\vec{a}$:
$$\Delta\vec{p} = \vec{F}\Delta t$$
3. Every action has an equal and opposite reaction:
$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

Chapter 7 — Rotational Motion and the Law of Gravity

7.1 Measuring Ratational Motion

For an object moving in a circle with radius r through an arc length of s, the angle θ (in radians) swept by the object is:

$$\theta = \frac{s}{r}$$

The conversion between radians and degrees is:

$$\theta(\text{rad}) = \frac{\pi}{180^o}\theta(\text{deg})$$

The **angular displacement** ($\Delta\theta$) through which an object moves from θ_i to θ_f is, in rad:

$$\Delta\theta = \theta_f - \theta_i = \frac{s_f - s_i}{r} = \frac{\Delta s}{r}$$

The average **angular speed** (ω) of an object is, in $\frac{\text{rad}}{\text{s}}$, the ratio between the angular displacement and the time interval required for that displacement:

$$\omega_{avg} = \frac{\theta_f - \theta_i}{\Delta t} = \frac{\Delta\theta}{\Delta t}$$

The average **angular acceleration** (α) is, in $\frac{\text{rad}}{\text{s}^2}$:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\Delta\omega}{\Delta t}$$

For each quantity or relationship in linear motion there is a corresponding quantity or relationship in angular motion:

Linear	Angular
x	θ
v	ω
a	α
$v_f = v_i + a\Delta t$	$\omega_f = \omega_i + \alpha\Delta t$
$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$	$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$
$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$	$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$

7.2 Tangential and Centripetal Acceleration

In addition to the angular speed of an object moving with circular motion it is also possible to measure the object's **tangential speed** (v_t) or instantaneous linear speed which is measured in $\frac{\text{m}}{\text{s}}$ as follows:

$$v_t = r\omega$$

An object's **tangential acceleration** (a_t) can also be measured, in $\frac{\text{m}}{\text{s}^2}$, as follows:

$$a_t = r\alpha$$

The **centripetal acceleration** (a_c) of an object is directed toward the center of the object's rotation and has the following magnitude:

$$a_c = \frac{v_t^2}{r} = r\omega^2$$

7.3 Causes of Circular Motion

The **centripetal force** (F_c) causing a centripetal acceleration is also directed toward the center of the object's rotation, and has the following magnitude:

$$F_c = ma_c = \frac{mv_t^2}{r} = mr\omega^2$$

The centripetal force keeping planets in orbit is a **gravitational force** (F_g) and it is found with Newton's Universal Law of Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

where G is the **constant of universal gravitation** which has been determined experimentally to be

$$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

An object in a circular orbit around the Earth will satisfy the equation

$$v = \sqrt{\frac{GM_E}{r}}$$

where M_E is the mass of the Earth.

Chapter 8 — Rotational Equilibrium and Dynamics

8.1 Torque

Just as a net external force acting on an object causes linear acceleration, a net **torque** (τ) causes angular acceleration. The torque caused by a force acting on an object is:

$$\tau = rF\sin\theta$$

where F is the force causing the torque, r is the distance from the center to the point where the force acts on the object, and θ is the angle between the force and a radial line from the object's center through the point where the force is acting. Torque is measured in $\mathrm{N}\cdot\mathrm{m}$.

The convention used by the book is that torque in a counterclockwise direction is positive and torque in a clockwise direction is negative (this corresponds to the right-hand rule). If more that one force is acting on an object the torques from each force can be added to find the net torque:

$$\tau_{net} = \sum \tau$$

8.2 Rotation and Inertia

The **center of mass** of an object is the point at which all the mass of the object can be said to be concentrated. If the object rotates freely it will rotate about the center of mass.

The **center of gravity** of an object is the point through which a gravitational force acts on the object. For most objects the center of mass and center of gravity will be the same point.

The **moment of inertia** (I) of an object is the object's resistance to changes in rotational motion about some axis. Moment of inertia in rotational motion is analogous to mass in translational motion.

Some moments of inertia for various common shapes are:

Shape	I
Point mass at a distance r from the axis	mr^2
Solid disk or cylinder of radius r about the axis	$\frac{1}{2}mr^2$
Solid sphere of radius r about its diameter	$\frac{2}{5}mr^2$
Thin spherical shell of radius r about its diameter	$\frac{2}{3}mr^2$
Thin hoop of radius r about the axis	mr^2
Thin hoop of radius r about the diameter	$\frac{1}{2}mr^2$
Thin rod of length l about its center	$\frac{1}{12}ml^2$
Thin rod of length l about its end	$\frac{1}{3}ml^2$

An object is said to be in **rotational equilibrium** when there is no net torque acting on the object. If there is also no net force acting on the object (**translational equilibrium**) then the object is in **equilibrium** (without any qualifying terms).

Type	Equation	Meaning
Translational Equilibrium	$\sum \vec{F} = 0$	The net force on the object is zero
Rotational Equilibrium	$\sum \tau = 0$	The net torque on the object is zero

8.3 Rotational Dynamics

Newton's second law can be restated for angular motion as:

$$\tau_{net} = I\alpha$$

This is parallel to the equation for translational motion as follows:

Type of Motion	Equation
Translational	$\vec{F} = m\vec{a}$
Rotational	$\tau = I\alpha$

Just as moment of inertia was analogous to mass, the **angular momentum** (L) in rotational motion is analogous to the momentum of an object in translational motion.

This is parallel to the equation for translational motion as follows:

Type of Motion	Equation
Translational	$\vec{p} = m\vec{v}$
Rotational	$L = I\omega$

The angular momemtum of an object is conserved in the absence of an external force or torque.

Rotating objects have **rotational kinetic energy** according to the following equation:

$$KE_r = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

Just as other types of mechanical energy may be conserved, rotational kinetic energy is also conserved in the absence of friction.

8.4 Simple Machines

There are six fundamental types of machines, called **simple machines**, with which any other machine can be constructed. They are levers, inclined planes, wheels, wedges, pulleys, and screws.

The main purpose of a machine is to magnify the output force of the machine compared to the input force, the ratio of these forces is called the **mechanical advantage** (MA)

of the machine. It is a unitless number according to the following equation:

$$MA = \frac{output\ force}{input\ force} = \frac{F_{out}}{F_{in}}$$

When frictional forces are accounted for, some of the output force is lost, causing less work to be done by the machine than by the original force. The ratio of work done by the machine to work put in to the machine is called the **efficiency** (*eff*) of the machine:

$$eff = \frac{W_{out}}{W_{in}}$$

If a machine is perfectly efficient ($eff = 1$) then the **ideal mechanical advantage** (*IMA*) can be found by comparing the input and output distances:

$$IMA = \frac{d_{in}}{d_{out}}$$

This leads to another way to find the efficiency of the machine as well:

$$eff = \frac{MA}{IMA}$$

Chapter 12 — Vibrations and Waves

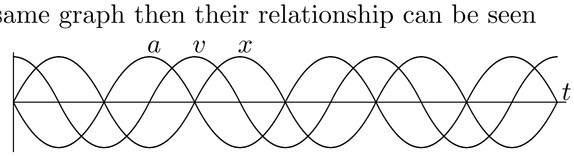
12.1 Simple Harmonic Motion

Hooke's law states that for a given spring the force the spring exerts ($\vec{F}$) is proportional to the negative of the displacement that the spring is stretched from rest ($\vec{x}$), or

$$\vec{F} = -k\vec{x}$$

Simple harmonic motion is the repetitive, back-and-forth motion of an object such as a pendulum or a mass oscillating on the end of a spring. The motion is over the same path each time and passes through the equilibrium position twice each cycle under the influence of a restoring force (a force that pushes the object back toward the equilibrium position).

The displacement (x), velocity (v), and acceleration (a) of an object in simple harmonic motion when graphed with respect to time are all sinusoids, if they are all plotted on the same graph then their relationship can be seen



12.2 Measuring Simple Harmonic Motion

The **amplitude** of harmonic motion measures how far the object moves.

The **period** (T) is the time that it takes for one complete cycle of motion.

The **frequency** (f) is the number of complete cycles of motion occurring in one second. Frequency is measured in inverse seconds (s^{-1}) or Hertz (Hz). Frequency and period are inverses of each other, so:

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

For a simple pendulum of length L, the period of small oscillations is given by:

$$T_{pendulum} = 2\pi\sqrt{\frac{L}{g}}$$

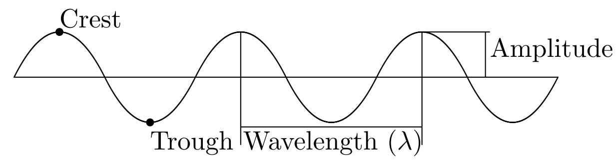
For a mass-spring system with mass m and spring constant k, the period of oscillation is:

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

12.3 Properties of Waves

Mechanical waves require the presence of a **medium** through which to move, such as a ripple traveling on the surface of a pond, or a pulse traveling along a stretched spring. If the vibrations of the wave are perpendicular to the direction of motion of the wave then it is a **transverse wave**, if the vibrations are parallel to the direction of motion then it is a **longitudinal wave**.

The displacement of a wave moving with simple harmonic motion can be modeled by a sinusoidal wave.



The **wavelength** (λ) of a wave is the shortest distance between corresponding parts of two waves, such as from one **crest** or **trough** to the next one.

The **wave speed** (v) is the speed with which a wave propagates, it is equal to the frequency of the wave times its wavelength:

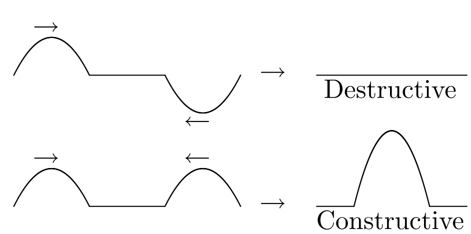
$$v = f\lambda = \frac{\lambda}{T}$$

The energy transferred by a wave is proportional to the square of its **amplitude**.

12.4 Wave Interactions

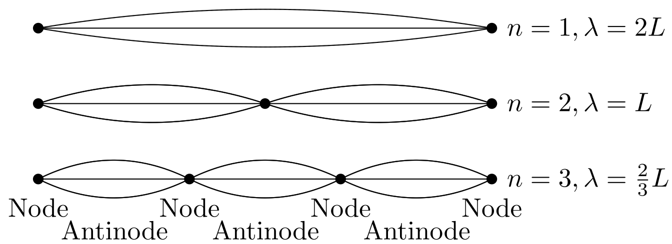
When two (or more) waves come together they pass through each other, and through the principle of **superposition** the total displacement of the medium is equal to the sum of the displacements of the overlapping waves at each point.

When two waves overlap with displacements in the same direction the resulting wave has an amplitude greater than either of the two overlapping waves, this is called **constructive interference**. In the other case, when two overlapping waves have displacements in opposite directions the resulting wave will have a displacement less than the displacement of the wave with larger amplitude, possibly even no displacement at all. This is called **destructive interference**.



When a wave traveling through some medium reaches a fixed boundary (a transition to a medium with a smaller, possibly zero, wave speed) the wave will reflect back inverted with respect to the initial wave. If the wave instead reflects off of a free boundary (a transition to a medium with a greater wave speed) then the wave will reflect off the boundary in the same orientation that it arrived.

If a wave is traveling in a confined space with just the right frequency then a **standing wave** may occur. A standing wave results from a wave constructively and destructively interfering with its own reflection. There will be some points (at least two, the ends) called **nodes** where complete destructive interference occurs, and between every pair of nodes there will be an **antinode** where constructive interference causes the oscillation to reach a relative maximum amplitude.



For a vibrating spring or string of length L the wavelengths of possible standing waves are:

$$\lambda = 2L, \quad L, \quad \frac{2}{3}L, \quad \frac{1}{2}L, \quad \frac{2}{5}L \ldots \qquad \lambda = \frac{2}{n}L$$

where n is the number of the harmonic that corresponds to that standing wave. The nth harmonic will have $n + 1$ nodes and n antinodes.

n	Nodes	Antinodes	λ
1	2	1	$\lambda = 2L$
2	3	2	$\lambda = L$
3	4	3	$\lambda = \frac{2}{3}L$
4	5	4	$\lambda = \frac{1}{2}L$
5	6	5	$\lambda = \frac{2}{5}L$
		...	
n	$n + 1$	n	$\lambda = \frac{2}{n}L$

Chapter 13 — Sound

13.1 Sound Waves

Sound waves are longitudinal waves consisting of successive **compressions** (areas of high pressure) and **rarefactions** (areas of low pressure) caused by and radiating outward in spherical wavefronts from a vibrating object.

The frequency a sound wave is perceived to have is called the **pitch**.

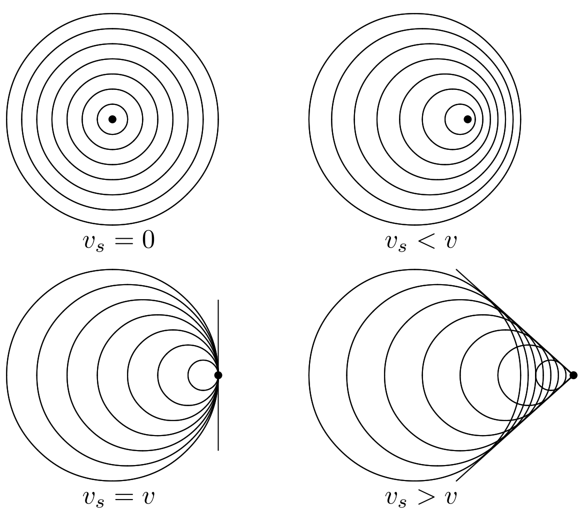
Sound waves travel outward in 3 dimensions along spherical wavefronts. If a ray is drawn from the source out through the wavefronts a graph of pressure vs. distance will be a sinusoid. When $r \gg \lambda$ the wave fronts begin to look like and can be approximated by plane waves.

Humans can hear sounds with frequencies from about 20 Hz to about 20,000 Hz with the greatest sensitivity in the range of 500 Hz to 5000 Hz. Frequencies outside that range are called either **infrasonic** (below 20 Hz) or **ultrasonic** (above 20,000 Hz).

The speed at which a sound wave propagates depends on the medium. In air at sea level and room temperature the speed of sound is approximately 343 $\frac{\text{m}}{\text{s}}$.

Doppler Effect

If a sound source and the detector are moving relative to one another then the **Doppler effect** can be observed.



This is a change in the pitch (f') relative to the frequency as emitted by the source (f). For a wave traveling at speed v, a detector moving at velocity v_d (positive if toward the source), and a source moving at velocity v_s (positive if toward the detector), the pitch will be

$$f' = f\left(\frac{v + v_d}{v - v_s}\right)$$

13.2 Sound Intensity and Resonance

Intensity

As a sound wave travels through a medium energy is transferred from one molecule to the next. The rate at which this energy is transferred is called the **intensity** and can be described as power per unit area and measured in watts per square meter.

In a spherical wave the energy is spread across the surface of a sphere, so the expression for intensity becomes

$$\textit{intensity} = \frac{P}{4\pi r^2}$$

The quietest sound audible to the human ear (the **threshold of hearing**) is about $1.0 \times 10^{-12}\ \frac{\text{W}}{\text{m}^2}$. The loudest sound a typical person can tolerate (the **threshold of pain**) is about $1.0\ \frac{\text{W}}{\text{m}^2}$.

Perceived Intensity

The perceived loudness of a sound is dependent on the logarithm of the intensity, so it is measured in decibels (dB). The sound level measured in decibels is:

$$\text{dB} = 10\log_{10}\frac{\textit{intensity}}{\textit{intensity}_r}$$

where *intensity* is the intensity of the sound being measured and $\textit{intensity}_r$ is the reference intensity. If nothing else is specified the reference intensity is set equal to the threshold of hearing or $\textit{intensity}_r = 1.0 \times 10^{-12}\ \frac{\text{W}}{\text{m}^2}$.

An increase of 10 decibels will be perceived as being twice as loud but will cause the intensity to increase by a factor of ten.

Resonance

Just as was observed with standing waves in a spring, certain physical systems show a particular response at certain frequencies called **resonance**. When a system is driven at (or very close to) its **resonant frequency** the amplitude of its vibration will continue to increase.

13.3 Harmonics

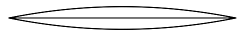
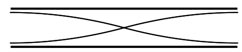
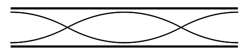
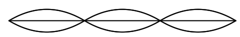
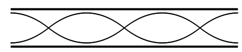
Standing Waves

Strings and air-filled pipes can exhibit standing waves at certain frequencies. The lowest frequency able to form a standing wave is called the **fundamental frequency** (f_1), those above it are **harmonics** of that frequency. The frequency of each harmonic frequency is equal to the fundamental frequency multiplied by the harmonic number (n), this forms a **harmonic series**. The fundamental frequency is also referred to as the first harmonic frequency.

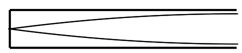
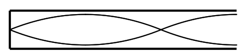
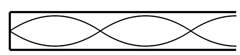
Depending on the medium in which the standing waves are formed the characteristics of the waves will vary. A pipe open at both ends has the same harmonic series as a vibrating string (producing all of the harmonics), while a pipe closed at one end will only produce odd-numbered harmonics.

The harmonics that are present form a sound source will affect the **timbre** or sound quality of the resulting sound.

Standing Waves on a String or Open Pipe of Length L

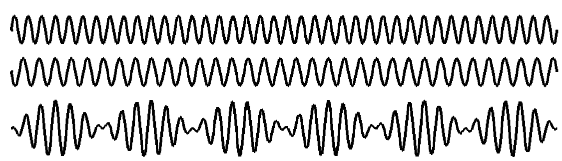
n	Nodes	Antinodes	String Diagram	Pipe Diagram	Wavelength	Frequency
1	2	1			$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$
2	3	2			$\lambda_2 = L$	$f_2 = \frac{v}{L} = 2f_1$
3	4	3			$\lambda_3 = \frac{2}{3}L$	$f_3 = \frac{3v}{2L} = 3f_1$
n	$n+1$	n	$n = 1, 2, 3, \ldots$		$\lambda_n = \frac{2}{n}L$	$f_n = n\frac{v}{2L} = nf_1$

Standing Waves in a Pipe of Length L Closed at One End

n	Nodes	Antinodes	Diagram	Wavelength	Frequency
1	1	1		$\lambda_1 = 4L$	$f_1 = \frac{v}{4L}$
3	2	2		$\lambda_3 = \frac{4}{3}L$	$f_3 = \frac{3v}{4L} = 3f_1$
5	3	3		$\lambda_5 = \frac{4}{5}L$	$f_5 = \frac{5v}{4L} = 5f_1$
n	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$n = 1, 3, 5, \ldots$	$\lambda_n = \frac{4}{n}L$	$f_n = n\frac{v}{4L} = nf_1$

Beat Frequency

If two sources emit sound waves that are close to one another in frequency but not exactly the same then a **beat** will be observed. When the amplitudes of the waves are added together the resulting sound oscillates between loud and soft. The number of beats heard per second is equal to the absolute value of the difference between the frequencies of the two sounds. The diagram below shows one second of the sum of a 34 Hz wave and a 40 Hz wave yielding a 6 Hz beat frequency.



$f = 40$ Hz

$f = 34$ Hz

Sum

Chapter 14 — Light and Reflection

14.1 Characteristics of Light

Light is an **electromagnetic wave** as opposed to a mechanical wave, so it does not need a medium through which to travel although it can pass through many media. Being an electromagnetic wave it is composed of a vibrating electric field and a vibrating magnetic field, the two fields are at right angles to each other.

The speed of light depends on the medium through which it travels, it is fastest in a vacuum and slower in denser media. The speed of light in a vacuum is exactly 299 792 458 $\frac{\text{m}}{\text{s}}$ (the definition of a meter comes from the speed of light so this is defined as the exact value). For most purposes, a value of 3.00×10^8 $\frac{\text{m}}{\text{s}}$ is sufficiently precise. The speed of light in a vacuum is a fundamental constant in many calculations and has been assigned the symbol c, thus for light and other electromagnetic waves the wave speed equation $v = f\lambda$ becomes:

$$c = f\lambda$$

Visible light is only one part of the **electromagnetic spectrum** corresponding to wavelengths from 400 nm (violet light) to 700 nm (red light).

Huygens' principle states that all of the points on a wave front can be treated as point sources and each of those point sources will produce a circular or spherical **wavelet**. The wavelets will constructively interfere on a line tangent to the fronts of all of the wavelets, this will create the next wave front. This principle will become more important later when we discuss diffraction, for now we can use the **ray approximation** which states that light can be considered to be straight lines traveling outward from the source perpendicular to the wave fronts.

The brightness of a surface lit by a single light is inversely proportional to the square of the distance between the source and the surface, for a source twice as far away the surface is one-fourth as bright, etc.

14.2 Flat Mirrors

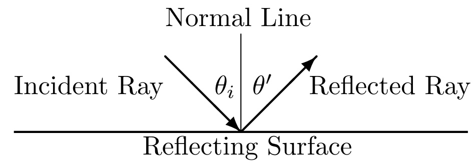
Reflection

Light traveling through a uniform medium will move in a straight line regardless of the medium.

When light encounters a boundary between media part of the light will pass through, part will be absorbed, and part will be bounced off of the boundary, or **reflected**. If the medium is one that is opaque to the light then none of it will pass through, the light will either be absorbed or reflected instead.

If the surface is rough the result is **diffuse reflection**, if it's smooth then it's **specular reflection**. Smooth vs. rough depends on the relative sizes of the surface variations and λ.

The light ray coming to the surface is the **incident ray**, the ray bouncing off of it is the **reflected ray**, and a line perpendicular to the surface at the point where the incident ray strikes it is the **normal line**. The angle between the incident ray and the normal line is the **angle of incidence** (θ_i) and the angle between the normal line and the reflected ray is the **angle of reflection** (θ'). The angle of incidence is always equal to the angle of reflection ($\theta_i = \theta'$).

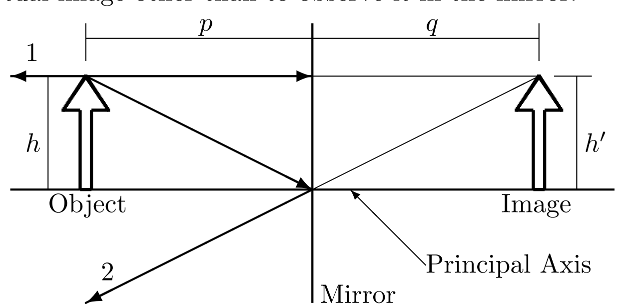


Flat Mirrors

If you put an object at some distance in front of a flat mirror (the **object distance,** p) light from the object will reflect off of the mirror and appear to come from a location behind the mirror's surface. As a convention, the object's image is said to be at that location (the **image distance,** q) behind the mirror. For a flat mirror the object is the same distance in front of the mirror as the image is behind it and the height of the image and object are equal ($h' = h$).

Ray Diagrams

Ray diagrams are drawn to locate the image formed by a mirror. To simplify matters we just consider a single point on an object resting on the **principal axis**. Rays are drawn from the tip of the object to the mirror and allowed to reflect back. In the diagram below ray 1 is drawn perpendicular to the mirror, it reflects back along itself and away. Ray 2 is drawn from the object to the mirror at the principal axis where it reflects back out at the same angle. The two rays as drawn do not meet to form an image so the need to be extended backward behind the mirror to where the image will be formed. The rays extended backward are not actually there but they seem to originate at the source of the image. Rays that are not there but seem to be are called **virtual rays** (**real rays** are the rays that are actually there) and the image that they form is a **virtual image** as opposed to a **real image** that is formed by real rays. A real image could be projected onto a screen, there is no way to see a virtual image other than to observe it in the mirror.



Ray diagram of a virtual image formed by a flat mirror

14.3 Curved Mirrors

Curved mirrors can be either **concave** (curving inward in the middle) or **convex** (curving outward in the middle). They can be approximated as a section of a sphere with radius of curvature R, and will have a **focal length** (the distance from the center of the mirror to the **focal point**, F) of $f = R/2$. By convention f and R are positive for a concave mirror and negative for a convex mirror. Similarly, p and q are considered to be positive if they are measured to a point in front of the mirror and negative if the point is behind the mirror. For a flat mirror $f = R = \infty$.

For a given mirror of focal length f, the relationship between focal length, object distance (p), and image distance (q) is:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

For an object of height h and an image of height h' their ratio is the **magnification**, M and is given as:

$$M = \frac{h'}{h} = -\frac{q}{p}$$

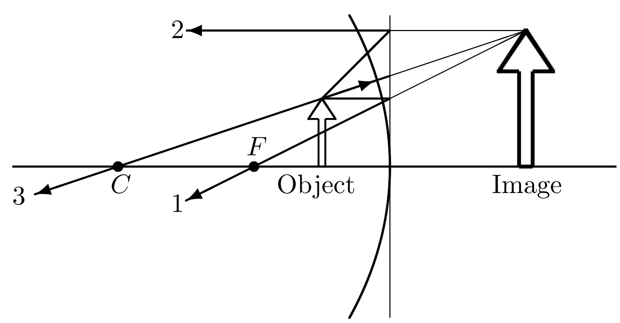
Sign conventions for mirrors

Symbol	Quantity	Situation	Sign
p	Object distance	Object is in front of the mirror	$+$
q	Image distance	Image is in front of the mirror (real image)	$+$
		Image is behind the mirror (virtual image)	$-$
h	Object height	Object is above the axis	$+$
h'	Image height	Image is above the axis	$+$
		Image is below the axis	$-$
f	Focal length	Focal point is in front of the mirror (concave mirror)	$+$
		Focal point is behind the mirror (convex mirror)	$-$
		No focal point (flat mirror)	∞
R	Radius of Curvature	Center of curvature is in front of the mirror (concave mirror)	$+$
		Center of curvature is behind the mirror (convex mirror)	$-$
		No center of curvature (flat mirror)	∞

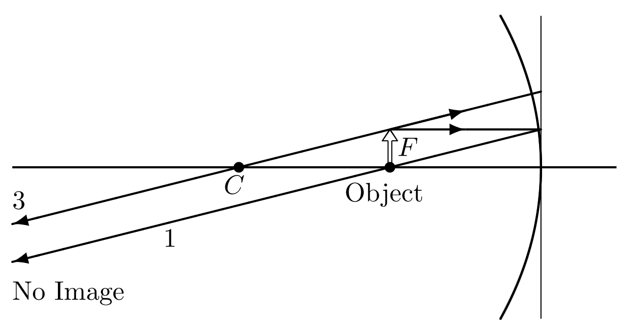
Images can be located through ray diagrams as well. The general principle is the same as the ray diagram for the flat mirror, the usual rays to be drawn are:

1. A ray that comes in parallel to the principal axis and goes out through the focal point.
2. A ray that comes in through the focal point and goes out parallel to the principal axis.
3. A ray that goes in through the center of curvature, reflects perpendicularly off of the mirror, and goes back out along the same line where it came in.

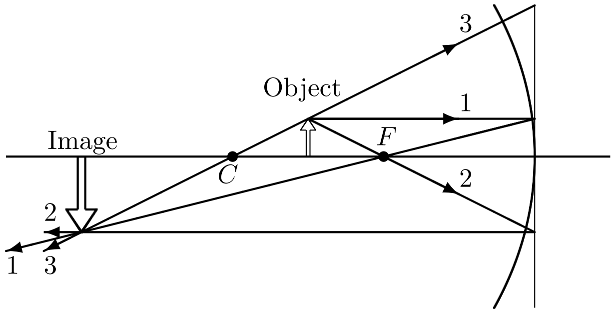
Each of these rays is labeled with its corresponding number in the ray diagrams below. For more precise results the rays are drawn to a line perpendicular to the principal axis rather than to the mirror itself.



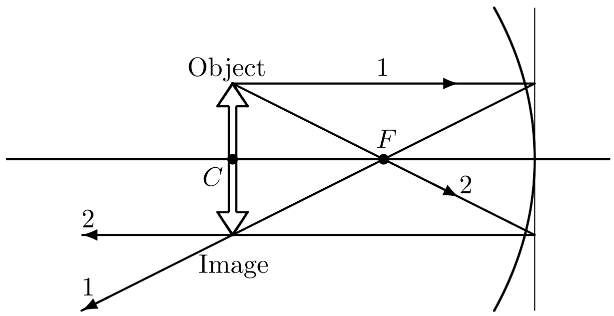
Concave mirror, $p < f$, virtual upright image, $M > 1$



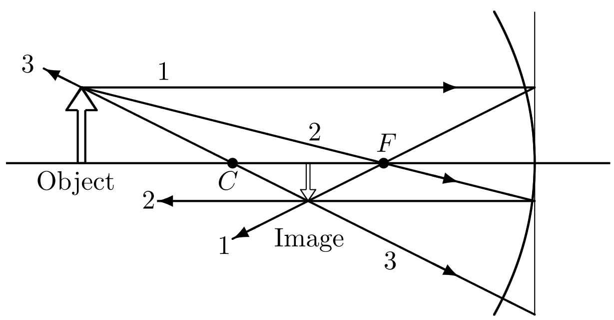
Concave mirror, $p = f$, no image



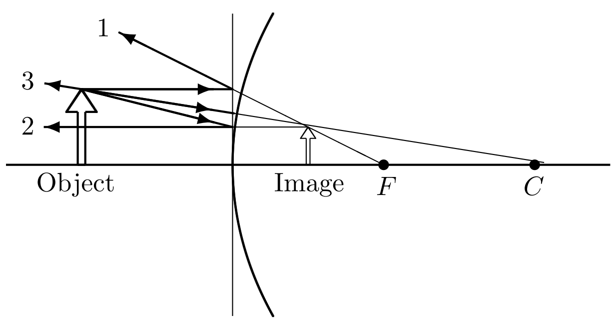
Concave mirror, $f < p < R$, real inverted image, $|M| > 1$



Concave mirror, $f = R$, real inverted image, $|M| = 1$



Concave mirror, $f > R$, real inverted image, $|M| < 1$



Convex mirror, virtual upright image, $M < 1$

Parabolic Mirrors

Spherical mirrors come close to focusing incoming parallel rays to a single point, particularly if all of the rays strike close to the center of the mirror. The farther the rays strike from the center the greater the error in focusing at a single point, this is called **spherical aberration**. If a parabolic mirror is used instead of a spherical mirror the spherical aberration is eliminated. High-quality optical instruments use parabolic mirrors for their improved optical properties.

14.4 Color and Polarization

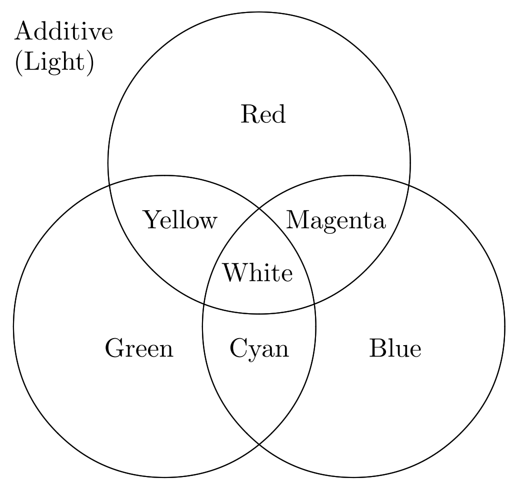
Color

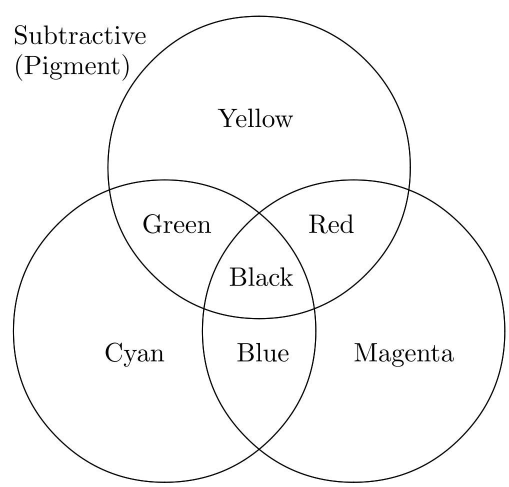
The primary colors of light are red, green, and blue. Colors of light mix through additive mixing, when all three primary colors are mixed white light is produced. The primary colors of pigment are cyan, magenta, and yellow. Colors of pigment mix through subtractive mixing, when all three primary colors are mixed black is produced. The primary colors for additive mixing are the secondary colors for subtractive mixing and vice versa.

Pigments appear to be the color that they are because that color of light is reflected, all other colors are absorbed. When two pigments are mixed the resulting mix will absorb the colors absorbed by both pigments that were mixed and only reflect the colors that neither pigment will absorb.

Primary Colors

In the diagrams below the circles represent the primary colors and the overlapping areas the result of mixing those colors.





Additive and subtractive primary colors

Color	Additive (mixing light)	Subtractive (mixing pigment)
red	primary	complimentary to cyan
green	primary	complimentary to magenta
blue	primary	complimentary to yellow
cyan (blue green)	complimentary to red	primary
magenta (red blue)	complimentary to green	primary
yellow	complimentary to blue	primary

Polarization

Light can be **polarized**, or set to vibrate in the same planes (one for electric field, one for magnetic field), by passing it through certain materials.

If a polarized beam of light is passed through a second polarizing filter only the component of that light matching the polarization of the new filter will pass through, so the overall intensity of the light will be proportional to the cosine of the angle between the polarizing filters.

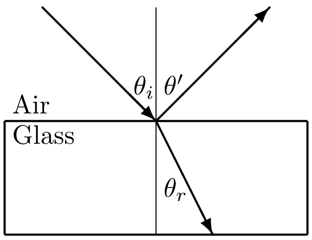
Light can be polarized by reflection at certain angles (about 34° up from the surface of a glass plate works well, the best angle varies depending on the material). Polarization is perpendicular to the plane containing the rays or horizontal for reflection off of a level surface. A pair of sunglasses with vertical polarization will remove the polarized glare from reflecting surfaces on a sunny day.

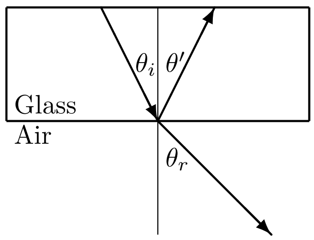
Chapter 15 — Refraction

15.1 Refraction

As light travels from one medium to another the speed of light also changes depending upon the densities of the media.

If a ray of light reaches a boundary between two media part of the ray will be reflected and part may also be **refracted** or bent while passing across the boundary.





The **index of refraction** (n) of a medium is a measure of how much the speed of light changes in that medium compared to in a vacuum. For a medium with speed of light v, the index of refraction is:

$$n = \frac{c}{v}$$

The amount that light will be refracted at a boundary depends on the respective indices of refraction of the media. All angles are measured from a line normal to the boundary at the point that the light intersects the boundary. For an **angle of incidence** θ_i in a medium with index of refraction n_i and an **angle of refraction** θ_r in a medium with index of refraction n_r the relationship is given by Snell's law:

$$n_i \sin\theta_i = n_r \sin\theta_r$$

15.2 Thin Lenses

Lenses operate similarly to curved mirrors with a few important differences: lenses have a focal point on both sides and some of the sign conventions are changed.

Sign conventions for lenses

Symbol	Positive (+)	Negative (−)
p	Object is in front of the lens	Object is in back of the lens
q	Image is in back of the lens	Image is in front of the lens
f	Converging lens	Diverging lens

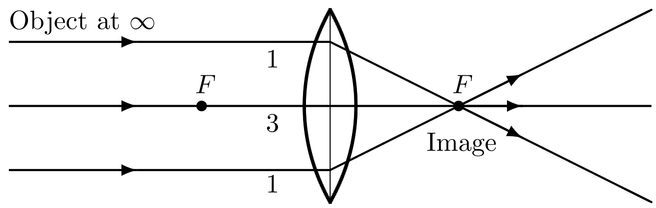
With those sign conventions in place the following equations still hold for focal length and magnification:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad M = \frac{h'}{h} = -\frac{q}{p}$$

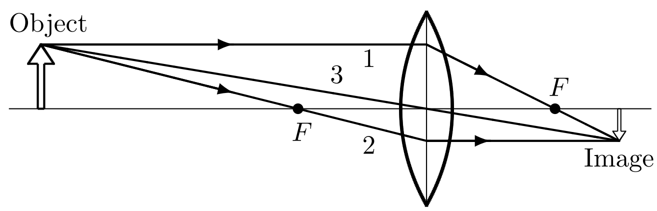
Ray diagrams can also be drawn to locate and describe the image that will be formed by a lens. The usual rays to be drawn are:

1. A ray that comes in parallel to the principal axis and goes out through the focal point.
2. A ray that comes in through the focal point and goes out parallel to the principal axis.
3. A ray that goes straight through the center of the lens

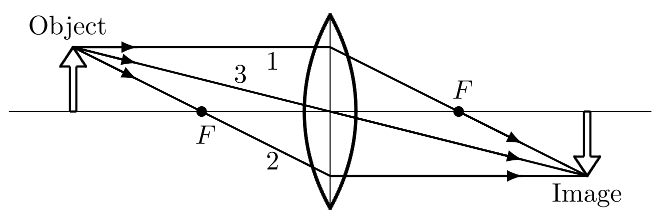
Each of these rays is labeled with its corresponding number in the ray diagrams below.



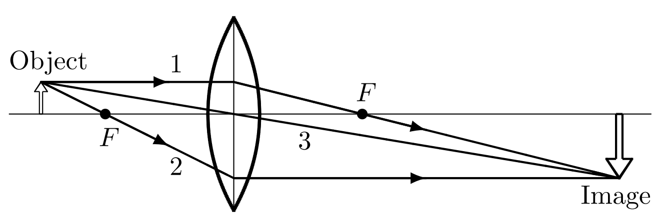
Converging lens, $p = \infty$, point image, $q = f$



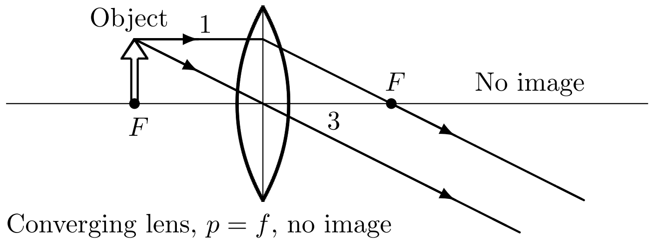
Converging lens, $p > 2f$, real image, $f < q < 2f$



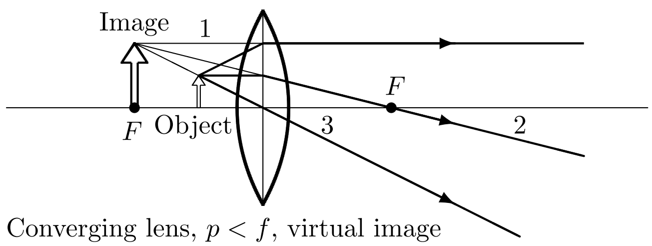
Converging lens, $p = 2f$, real image, $q = 2f$



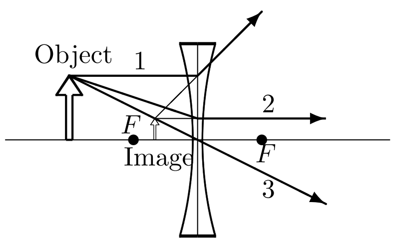
Converging lens, $f < p < 2f$, real image, $q > 2f$



Converging lens, $p = f$, no image



Converging lens, $p < f$, virtual image

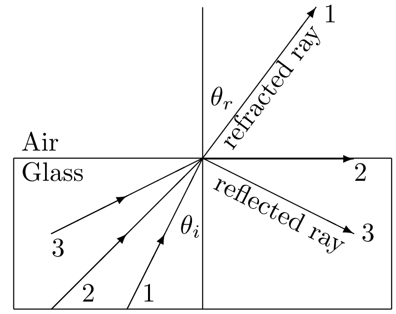


Diverging lens, virtual image

Lenses can be used for the correction of defects in human vision. If a person has **hyperopia** (farsightedness) light rays tend to focus behind the retina, a converging lens will cause them to focus on the retina. Similarly, if a person has **myopia** (nearsightedness) a diverging lens will cause the light rays to focus on the retina instead of in front of it.

Lenses can be used in combination, the image formed by the first lens is treated as the object for the second lens. The overall magnification of the system is the product of the magnification of the individual lenses.

15.3 Optical Phenomena



When light passes from a medium with a higher index of refraction to one with a lower index of refraction an effect called **total internal reflection** can sometimes be observed. If the angle of incidence is greater than some **critical angle** θ_c then there will be no refracted ray. The expression for that critical angle is:

$$\sin\theta_c = \frac{n_r}{n_i} \quad \text{for } n_i > n_r$$

The diagram above shows incident rays at less than the critical angle (1), equal to the critical angle (2), and greater than the critical angle (3). All three incident rays would produce reflected rays but they have been omitted from 1 and 2 for clarity.

Refraction can occur even if there is not an abrupt boundary between the two media, if for example warm (less dense) air is trapped below cooler (more dense) air then light rays passing through the air will be refracted slightly. It is very difficult to predict the exact amount of refraction that will be observed.

When light travels through the atmosphere it is scattered with the most scattering happening to the shorter wavelengths of light, this causes the sky to appear to be blue and sunsets, after the sunlight has had most of the blue scattered when traveling through the atmosphere, to appear orange or red. The name for this phenomenon is **Rayliegh scattering**.

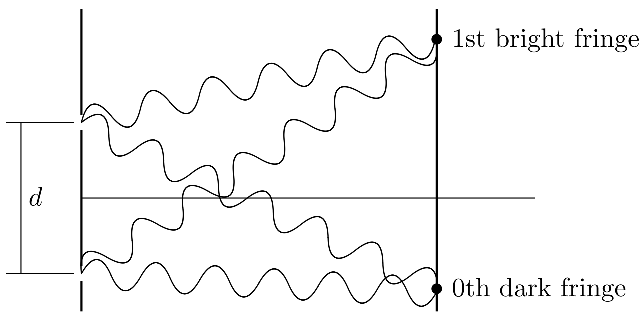
The index of refraction in a medium is dependent on the wavelength of light involved, because of this a boundary between media may refract different wavelengths by different amounts, a phenomenon known as **dispersion**. This causes the display of a spectrum by a prism, rainbows, and **chromatic aberration** or the focusing of different colors of light by a lens at slightly different distances behind the lens.

Chapter 16 — Interference and Diffraction

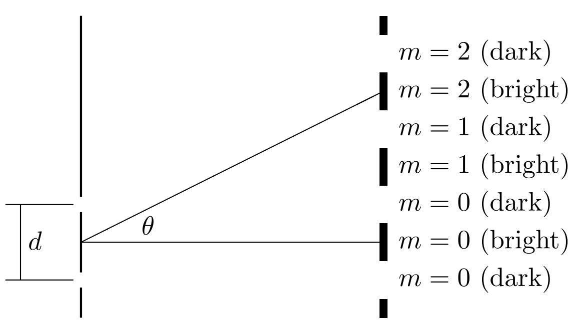
16.1 Interference

Interference can only happen between waves of the same wavelength. If the crests of two waves arrive at the same point then they have a phase difference of 0° and they are said to be **in phase**, if the crest of one wave matches a trough of the other then they have a phase difference of 180° and they are said to be **out of phase.**

If monochromatic light from a single source is passed through two narrow parallel slits they will serve as **coherent** sources and will produce an interference pattern with a bright **fringe** where constructive interference occurs and dark ones in between as shown below.



The fringes are numbered with an **order number,** the center one being assigned the number zero (the **zeroth-order maximum** or **central maximum**) and others numbering outward from there. The dark fringes are also numbered from zero although there are two zeroth-order dark fringes and only one bright one. The diagram below shows how the fringes produced by two slits a distance d apart are numbered (one side of the diagram is omitted to save space), the angle θ shown below corresponds to the 2nd bright fringe.



For two slits separated by a distance d the mth order maximum is located at some angle θ from a normal line drawn between the slits, that angle also depends on the wavelength of the light and can be found from the equation

$$d \sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, \pm3 \ldots$$

with the corresponding dark fringes being found from the similar equation

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm1, \pm2, \pm3 \ldots$$

If the difference in distances is an integral multiple of the wavelength then a bright fringe will be seen, if the difference is half a wavelength from an integral multiple of the wavelength then it will be a dark fringe.

When a white light source is used, instead of simple fringes a very different pattern is observed since each color of light refracts differently and the resulting fringes mix to form other colors.

16.2 Diffraction

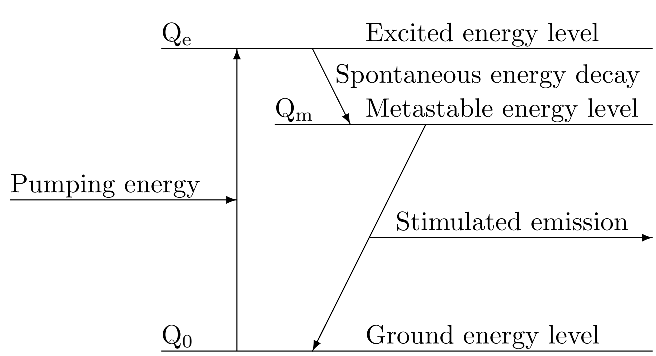
Since each point on a wave front is a source of Huygens wavelets interference patterns can be observed from a single slit. Light deviates from a straight-line path and enters the area that would normally be in shadow (**diffraction**) and produces a **diffraction pattern** when the wavelets interfere with each other. This can occur when light passes through a slit or when it passes by the edge of an object.

When light passes through a diffraction grating each adjacent pair of grooves in the grating will behave like the two slits described previously, the same equations will work to describe the location of the bright and dark fringes.

Diffraction plays a mayor part in determining the **resolving power** of an optical instrument. The resolving power is the instrument's ability to distinguish two objects that appear to be close to each other as separate images and it is measured as the minimum angle between the two objects (with the instrument at the vertex of the angle) that can be discerned. This angle is proportional to the wavelength of the light and inversely proportional to the size of the aperture through which the light must pass.

16.3 Lasers

In a typical light source the individual rays of light are emitted haphazardly, the waves do not share a common wavelength or phase.

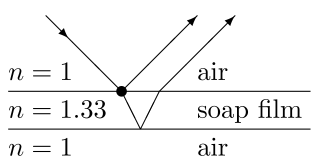


As shown above, lasers take advantage of properties of the energy levels of an electron in certain types of atoms. They do so by having a set of energy levels where some initial energy (the pumping energy) will raise an electron to an unstable, excited state. The electron will then spontaneously decay to a lower, metastable state (a state the electron can hold for some time) where it will remain until the electron is struck by a photon from another decaying electron, this will trigger the electron to drop back to the ground state and release an additional photon. Since all of the stimulated emission photons come from the same energy transition in the atoms they all have the same energy and thus frequency and wavelength. The waves are also produced in phase with each other, yielding what is called **coherent** light. The beam of a laser appears much more intense than a similar light source because of the coherence and the tendency of lasers to emit a nearly parallel beam of light (it does not spread very much from its source until it strikes an object).

Lasers are often used in information storage and retrieval (reading and writing CDs and DVDs), in measurement (finding the time for a laser to reflect from a distant object) and in medicine (precisely delivering energy to specific targets in the body that might not otherwise be reachable).

Thin-Film Interference

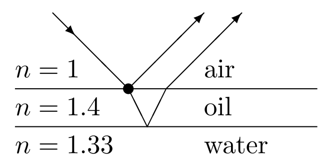
When light bounces off of a soap bubble it is reflected off of both the outside and inside surfaces. Depending on the exact thickness of the soap bubble the light reflecting off of one of the two surfaces will be either in or out of phase with the light reflecting from the other one. If they are in phase then a bright band will be observed, if not then it will be dark. To see this sort of interference the bubble must be at least $\frac{1}{4}\lambda$ thick, any thinner and the waves will be less than 180° out of phase.



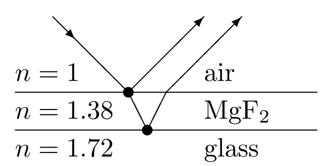
The black dot on the diagram above indicates that the phase of the reflected light at that point is shifted by 180° because the reflecting surface has a higher index of refraction. Since all other reflections are from a reflecting surface with a lower index of refraction they will not change the phase of the incident rays. If the thickness of the soap film is $\frac{1}{4}\lambda$ then there will be constructive interference, if it is $\frac{1}{2}\lambda$ then there will be descructive interference, and the pattern will repeat with each $\frac{1}{4}\lambda$ added to the thickness.

Since the bubbles vary in thickness from top to bottom the interference pattern observed will also vary. The pattern will generally consist of horizontal bands with some irregularities as the fluid in the bubble moves around. If the bubble is lit with only one color of light then the bands will be light and dark, if the bubble is lit with white light then a rainbow pattern will be observed.

Thin films of oil floating on water (as you may see on your driveway after it rains) exhibit similar properties, instead of seeing horizontal bands around a bubble you see concentric irregular rings of color as the thickness of the oil decreases toward the edges.



A similar principle controls the function of anti-reflection coatings on lenses, a very thin coating of magnesium fluoride is deposited on the surface of a glass lens to provide destructive interference for most wavelengths of visible light.



The main difference between the anti-reflection coatings and the soap or oil films is that there is an extra phase inversion at the coating-glass boundary. Because of this the interference at $\frac{1}{4}\lambda$ will be destructive and at $\frac{1}{2}\lambda$ it will be constructive.

Chapter 17 — Electric Forces and Fields

17.1 Electric Charge

Electric charge comes in two varieties, negative (an excess of electrons) and positive (a lack of electrons). Positive charges attract negative charges and vice versa, two like charges will repel each other.

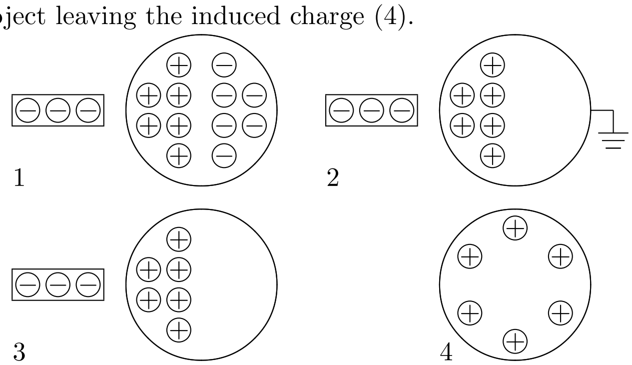
As shown by Robert Millikan in 1909 charge must come in multiples of a fundamental unit. This fundamental unit of charge, e, is the charge on a single electron or proton. The unit by which charge is usually measured is the coulomb (C), one coulomb is 6.2×10^{18} e and the charge on a single electron is -1.60×10^{-19} C.

Electric charge is conserved. It can me transferred from one object to another by moving electrons but it can not be created or destroyed.

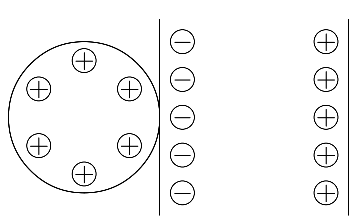
Materials can be either **conductors** (materials through which electrons flow freely), **insulators** (materials that do not permit the free flow of electrons) and **semiconductors** (materials that will either conduct or not depending on the circumstances).

Both conductors and insulators can be charged by **contact** (by placing two dissimilar objects together, the effect can be intensified by rubbing them together) although a conductor must not be connected to a ground for the effect to accumulate.

A conductor can also be charged by **induction** by bringing another charged object such as a rubber rod near it (1, below), allowing the displaced charge to drain off on the other side (2) then removing the drain while the charged object remains (3) then finally removing the original charged object leaving the induced charge (4).



A surface charge can be induced on an insulator by **polarization** (by bringing a charged object close to the surface, this will cause an opposite charge on the near surface and a charge the same as the charged object on the far side).



17.2 Electric Force

According to Coulomb's law, the electric force between two charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F_{electric} = k_C \frac{q_1 q_2}{r^2}$$

This is dependent on the Coulomb constant (k_C):

$$k_C = 8.987\ 551\ 788 \times 10^9\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \approx 8.99 \times 10^9\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

This is very similar to the equation for universal gravitation, substituting charge for mass and making corresponding changes in the constant.

Like gravity, the electric force is a field force so it does not require contact to act. The resultant electric force on any charge is the vector sum of the individual vector forces on that charge (the **superposition principle**). An time multiple forces act on a single charged object and the sum of the forces is zero the object is said to be in **equilibrium**.

17.3 The Electric Field

Electric Field Strength

A charged object sets up an **electric field** around it, when a second charge is present in this field an electric force will result. The strength of the electric field (E) is defined to be the force that would be experienced by a small positive **test charge** (q_0) divided by the magnitude of the test charge. Since the field strength is a force divided by a charge the units are newtons per coulomb ($\frac{N}{C}$). Mathematically, this can be expressed as

$$E = \frac{F_{electric}}{q_0}$$

Combining the above equation with Coulomb's law gives an equation that will give the electric field strength around a charge

$$E = \frac{k_C q}{r^2}$$

Since the test charge is always positive a positive charge will produce a field directed away from the charge, a negative charge will produce a field directed inward. The direction of the electric field vector, $\vec{E}$, is the direction in which an electric force would act on a positive test charge. The equation will provide the magnitude of the vector, the direction is found from the sign of the charge.

Electric Field Lines

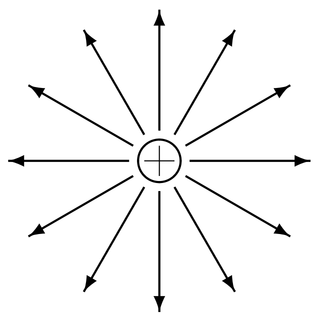
A convenient aid for visualizing electric field patterns is to draw **electric field lines**. They consist of lines drawn tangent to the electric field vector at any point, the number of lines drawn being proportional to the magnitude of the field strength. It is important to remember that electric field lines do not actually exist as drawn since the field is continuous at every point outside of a conductor; the field lines are only drawn as a representation of the field.

Rules for drawing field lines

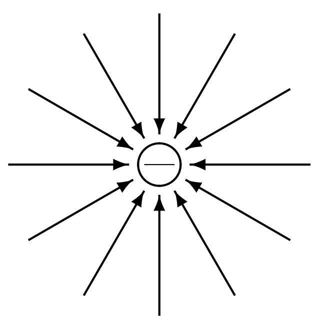
- Field lines must begin on positive charges or at infinity and must terminate on negative charges or at infinity.

- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

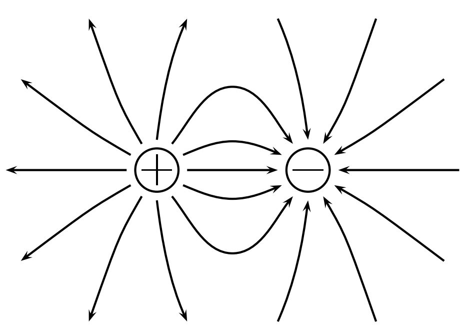
- No two field lines from the same field can cross each other.



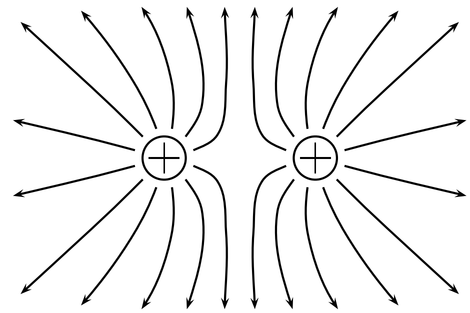
A single positive charge



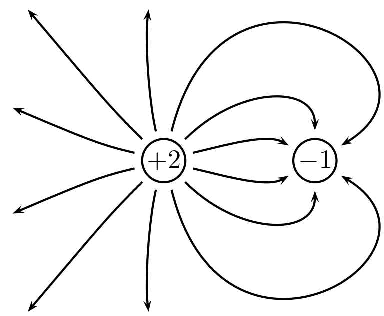
A single negative charge



Equal positive and negative charges



Equal positive charges



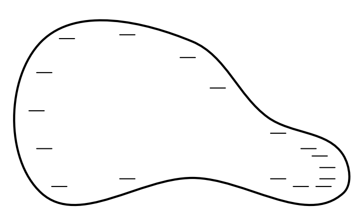
A postive charge twice the size of a negative charge

Conductors in Electrostatic Equilibrium

An electrical conductor such as a metallic object contains excess electrons (or a lack of them) that are free to move around the conductor. When no net motion of charge is occurring within the conductor it is said to be in **electrostatic equilibrium**. Any conductor in electrostatic equilibrium has the following properties:

- The electric field is zero everywhere inside the conductor.
- Any excess charge on an isolated conductor is entirely on the surface of the conductor.
- The electric field immediately outside of a charged conductor is perpendicular to the surface of the conductor.
- On an irregularly shaped conductor the charge will tend to accumulate where the radius of outward curvature is smallest. (This will produce the strongest electric fields just outside of those points.)

For example, consider this irregular conductor with an excess of electrons:

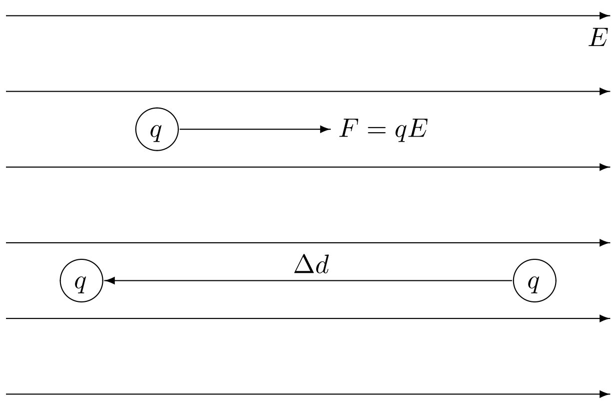


Chapter 18 — Electrical Energy and Capacitance

18.1 Electrical Potential Energy

A **uniform electric field** is an electric field that has all of the field lines parallel to and equidistant from each other. An electric field caused by a charge that is very far away is effectively uniform over a small area since the field lines are nearly parallel and the difference in strength over a small distance is negligible. Such a field could also be caused by a pair of oppositely charged plates, the field will be uniform in the region between the plates.

A uniform electric field will always exert a constant force on a charged object, if that object is displaced in the field then a force will be required causing work to be done and energy to be accumulated.



Just as gravitational potential energy was determined by taking the product of the weight of an object and its height from some reference height the same can be done for **electrical potential energy** ($PE_{electric}$) by taking the product of the force generated by the field and its position in the field. Mathematically, in a uniform electric field of strength E with a displacement d measured in the direction of E

$$PE_{electric} = -qEd$$

The negative sign is needed because the displacement of a positive charge to increase the potential energy should be in the direction opposite the direction of the field. If the initial and final positions are taken before and after a displacement the change in potential energy can be found thusly

$$\Delta PE_{electric} = -qE\Delta d$$

18.2 Potential Difference

Electric potential (V) is defined as the electrical potential energy per unit charge, so

$$V = \frac{PE_{electric}}{q} = \frac{-\cancel{q}Ed}{\cancel{q}} \rightarrow V = -Ed$$

The relative difference in electric potential between two points is the **potential difference** (ΔV) which, like electric potential, is measured in $\frac{\text{J}}{\text{C}}$ which are given the name **volts** (V).

$$\Delta V = \frac{\Delta PE_{electric}}{q} = -E\Delta d$$

Note that the electric potential at a given point is independent of the charge at that point and that it's only the electric potential difference between two points that is useful, the electric potential at a single point is based on an arbitrary reference similar to position.

18.3 Capacitance

Because of time constraints the topic of capacitance was omitted this year.

Chapter 19 — Current and Resistance

19.1 Electric Current

Current

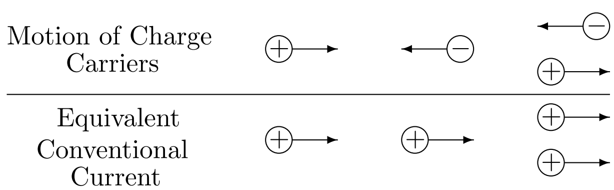
Electric **current** (I) is the rate at which positive charges move through a conductor past a fixed point. For a charge ΔQ moving during a time Δt the current is

$$I = \frac{\Delta Q}{\Delta t}$$

Current is measured in **amperes** (A) which are a fundamental SI unit from which coulombs are derived. ($1\ \mathrm{A} = 1\ \frac{\mathrm{C}}{\mathrm{s}}$ or more properly $1\ \mathrm{C} = 1\ \mathrm{A} \cdot \mathrm{s}$)

Charge Carriers

Electric charge can be either positive or negative, so either positive or negative charges could be carried by the particles moving to make up the current. The convention is to regard current as being the movement of positive charges, but in common conductors such as metals it is actually the negatively charged electrons that are moving. In such a case the current is considered to be positive charges moving in the opposite direction as shown below.



When studying current on a macroscopic scale there is no discernible difference between the actual motion of the charge carriers and the conventional current, it is when studying the charge carriers themselves that a distinction must be made.

Sources and Types of Current

The charge carriers (electrons, protons) are always present, all source of current does is move them to points with a potential difference between them and then allow them to flow back to a lower energy state through some device.

Batteries do this by converting chemical energy to electrical energy, generators do this by converting mechanical energy to electrical energy. Batteries always produce a fixed potential difference between their terminals (**direct current**, limited of course as the battery runs out of stored chemical energy), generators can either produce the same type of potential difference or one that reverses itself many times a second (**alternating current**). Alternating current has several practical advantages for energy transmission, we will address them later.

19.2 Resistance

The current through some circuit depends on the potential difference in the current source, a larger potential difference produces a larger current. If the circuit is replaced with a different one that can also change the current that will flow, some materials allow current to flow more easily than others. The opposition to the flow of current through a conductor is called **resistance** (R) which is defined to be

$$R = \frac{\Delta V}{I}$$

Resistance is measured in $\frac{\mathrm{V}}{\mathrm{A}}$ which are called ohms (Ω).

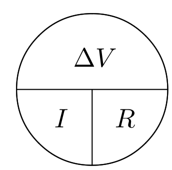
Ohm's Law

Ohm's law states that for many materials *the resistance is constant over a wide range of applied potential differences.* Mathematically

$$\frac{\Delta V}{I} = \text{constant}$$

Materials that follow this relationship are said to be **ohmic**, others (such as semiconductors) are **non-ohmic**. Materials may be ohmic at one temperature and non-ohmic at another.

An easy way to remember Ohm's law as well as to quickly derive it for any of the values is to show it this way:



If you put our finger on top of any one of the three quantities (ΔV, I, or R) the remaining two will be in the right position to tell you what to do with them to find the one you have covered. For example, to find current cover the I with your finger, what you then see is $\frac{\Delta V}{R}$ which is equal to the current.

R depends on length, cross-section, material, temperature. Longer conductors, smaller cross-sections, and higher temperatures all increase the resistance of a conductor and vice versa, replacing the material making up the conductor with another material will also change the resistance (for example, copper has lower resistance than aluminum).

Human Body Resistance

The resistance of the human body is about 500 000 Ω if the skin is dry, if the skin is soaked with salt water (or sweat) the resistance can drop to 100 Ω or so. A current through your body less than about 0.01 A is either imperceptible or felt as a slight tingling, greater currents are painful, and a current of about 0.15 A through the chest cavity can be fatal.

Superconductors

Some materials have zero resistance below a certain **critical temperature**, these materials are known as **super-**

conductors. The critical temperatures are generally very low (less than 10° K for most materials that superconduct) although some have been found that superconduct up to about 125° K.

19.3 Electric Power

Just as power was studied previously as the rate of conversion of mechanical energy it is also the rate at which electrical energy is transferred. Since

$$P = \frac{W}{\Delta t} = \frac{\Delta PE}{\Delta t} = \frac{q\Delta V}{\Delta t} \quad \text{and} \quad I = \frac{q}{\Delta t}$$

it follows that power is then

$$P = I\Delta V$$

or that power is the product of the current and the potential difference. Combining the equation for power with Ohm's law ($\Delta V = IR$) yields the relationships

$$P = I^2R \quad \text{and} \quad P = \frac{\Delta V^2}{R}$$

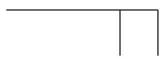
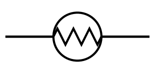
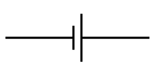
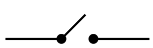
When electric utilities sell energy they use the derived unit kilowatt-hours (kW · h), one of which is defined to be the energy delivered at a constant rate of 1 kW in 1 h. A brief exercise in algebra will show that 1 kW · h = 3.6×10^6 J.

Since power converted by a resistive load is $P = I^2R$, the power lost to resistance in a wire is proportional to the square of the current. Reducing the current provides a considerable reduction in lost power so electrical transmission lines operate at very high potential differences to keep the current low, up to 765 000 V in the highest potential difference lines. Transformers are used to reduce that potential difference to 120 V close to the point where it will be used.

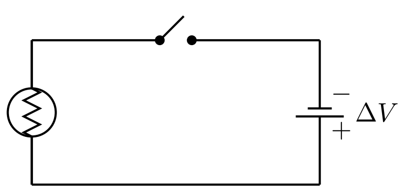
Chapter 20 — Circuits and Circuit Elements

20.1 Schematic Diagrams and Circuits

Electric circuits are generally drawn as **schematic diagrams** that show the electrical components and their connections but leave out unnecessary detail. There are several versions of the symbols used, the ones most commonly used by your textbook are:

Component	Symbol	Comments
Wire		Wires are represented by drawing lines between the elements to be connected
Resistor or load		All resistive loads share this symbol. If a distinction needs to be made it is generally made by adding a label.
Lamp		A lamp (or bulb) is shown as a resistor with a circle around it to represent the glass
Battery		The longer line represents the positive terminal.
Switch		An open switch is shown. A closed switch would bring the arm down to make contact on both sides.

A schematic diagram of a simple flashlight could look like this



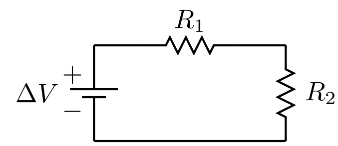
The battery is shown on the right, the lamp on the left, and a switch is at the top to allow the circuit to be opened or closed as desired. No details such as the flashlight housing, lens, or reflector are shown, they do not affect the electrical function of the flashlight so they are omitted.

The battery provides a potential difference at two points in the circuit, when the switch is open nothing will happen. When the switch is closed then the circuit is completed and the current can flow through the lamp, the amount of current flowing will depend on the potential difference provided by the battery and the resistance of the lamp.

20.2 Resistors In Series or In Parallel

Finding the current in a circuit that only contains a single resistor is simple, but finding the current in circuits with more than one resistor becomes more complex.

Resistors In Series



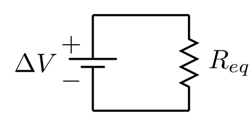
In the case where multiple resistors are connected in series the fact that the same current must flow through all of them leads to the equations

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

$$\Delta V_t = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$$

$$I_t = I_1 = I_2 = I_3 \ldots$$

which indicates that in a series circuit the equivalent resistance is equal to the sum of the individual resistances. All of the resistances can be replaced with a single resistor equal to their sum and the current will be the same

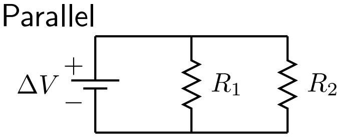


Once the current has been found through the equivalent resistance the potential difference across each resistor (ΔV_i) can be found by once again taking advantage of the fact the the current is the same through resistors in series and the sum of the individual potential differences is equal to the total potential difference, so

$$\Delta V_i = I_t R_i$$

If any element in a series circuit stops conducting then the flow of current through the entire circuit will stop as if a switch had been opened at that point.

Resistors In Parallel



For the case where multiple resistors are connected in series the fact that the potential difference across all of the resistors is the same and the total current must be the sum of the individual currents through each resistor leads to

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

$$\Delta V_t = \Delta V_1 = \Delta V_2 = \Delta V_3 \ldots$$

$$I_t = I_1 + I_2 + I_3 \ldots$$

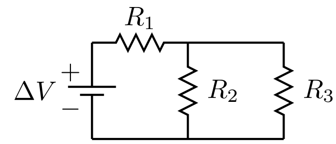
The current through each resistor (I_i) can be found with

$$I_i = \frac{\Delta V_t}{R_i}$$

Because of the reciprocal relationship used in finding the equivalent resistance for a parallel circuit the equivalent resistance will always be smaller than the smallest of the individual resistances.

If any element in a parallel circuit stops conducting then the flow of electricity will continue through the remaining parallel elements.

20.3 Complex Resistor Combinations



In a circuit like the one above with both parallel and series elements the methods to find each type of equivalent resistance must be applied repetitively until only one equivalent resistance is left. In this circuit R_1 is in series with the combination of R_2 and R_3 in parallel, so first the two parallel resistors R_2 and R_3 combine in parallel to give the equivalent resistance $R_{2,3}$, then $R_{2,3}$ and R_1 combine in series to give the overall equivalent resistance for the entire circuit.

Once the equivalent resistance has been found you work backward from there to find the current in and potential difference across each resistor.

Chapter 21 — Magnetism

21.1 Magnets and Magnetic Fields

Every magnet has two magnetic poles, one designated as north, one as south. A single magnet can be cut or broken into two pieces, each piece of a magnet will also be a magnet with two poles. (It is impossible to have a single magnetic pole without the corresponding opposite pole.) Like magnetic poles repel each other, unlike poles attract.

Just as an electric charge creates an electric field, a **magnetic dipole** (a pair of magnetic poles) creates a **magnetic field** around it and interacting with other magnetic dipoles in the vicinity.

The **magnetic field strength** (B) is measured in tesla (T) which are $\frac{\mathrm{N}}{\mathrm{A \cdot m}}$ (this is related to the force exerted on a moving charge).

The direction of a magnetic field is defined as the direction that the north pole of a magnet would point if placed in the field. The magnetic field of a magnet points from the north pole of the magnet to the south pole. By convention, when magnetic field lines are drawn they are represented as arrows when laying in the plane of the page, if they are coming up out of the page they are represented by a solid circle, and if they are going down into the page they are represented by an 'x'. It may help to think of an arrow like would be shot from a bow, if it is coming toward you then you will see the point, if it is moving away then you will see the feathers on the end.

The magnetic north pole of the earth corresponds to the geographic south pole and vice versa.

21.2 Electromagnetism and Magnetic Domains

A magnetic field exists around any current-carrying wire, the direction of the field around the wire follows a circular path around the wire according to the right-hand rule — if you point the thumb of your right hand in the direction of the current in the wire and curl your fingers the magnetic field will follow your fingers.

The magnetic field created by a solenoid or coil is similar to the magnetic field of a permanent magnet, the direction is once again determined by the right-hand rule — this time if you wrap the fingers of your right hand in the direction that the current flows in the coil then the magnetic field inside the coil will be in the direction of your thumb and will be nearly uniform within the coil.

A **magnetic domain** is a group of atoms whose magnetic fields are aligned. In some materials (wood, plastic, etc.) the magnetic fields of the individual atoms are arranged randomly and can not be aligned, others such as iron, cobalt, and nickel the magnetic fields do not completely cancel. These are known as **ferromagnetic** materials. When a substance becomes magnetized by an external magnetic field the individual domains are oriented to point in the same direction. If a material is easily magnetized then the domains will return to a random organization when the external field is removed, if it is not easily magnetized then the organization of the domains will remain when the field is removed and the substance will remain magnetized.

21.3 Magnetic Force

Charged Particles in a Magnetic Field

When a charge moves through a magnetic field it will experience a force caused by the magnetic field. This force has a maximum when the charge moves perpendicularly to the magnetic field, decreases at other angles, and becomes zero when the charge moves along the field lines.

The magnitude of the magnetic field (B) is given by

$$B = \frac{F_{magnetic}}{qv}$$

where q is the moving charge and v is its velocity.

The direction of the force on a positive charge moving through a magnetic field can be found using the right-hand rule. If you point the fingers in the direction of the magnetic field and your thumb in the direction of the motion of the charge and then bend your fingers at an angle the force will then be in the direction of your fingers. If the charge is negative then find the direction for a positive charge and then reverse it.

A charged particle moving perpendicularly to a uniform magnetic field will move in a circular path. If the particle is not moving perpendicularly to the magnetic field then the component of the particle's velocity in the direction of the field will remain unchanged and the particle will move in a helical path.

Magnetic Force on a Current-Carrying Conductor

A length of wire, l, within an external magnetic field, B, carrying a current, I, undergoes a magnetic force of

$$F_{magnetic} = BIl$$

Two parallel current-carrying wires exert on one another forces that are equal in magnitude and opposite in direction. If the currents are in the same direction the two wires attract one another, if they are in opposite directions then they repel each other.

The magnetic fields developed by conductors in an electric field are used to produce sound with loudspeakers and are also used to cause the needle to move in a galvanometer.

Chapter 22 — Induction and Alternating Current

22.1 Induced Current

Moving a conductor in an electric field or changing the magnetic field around a conductor will induce an *emf* (also known as an electromotive force, but it is actually a potential difference and not a force) in the conductor through **electromagnetic induction**. The charges already present in the conductor will be caused to move in opposite directions by the motion of the conductor through the field, thus a current can be induced without a battery or other source present. This effect is greatest when the motion of the conductor is perpendicular to both the conductor and the magnetic field.

If a loop (or coil) of wire is moved linearly through a magnetic field the orientation of the loop will affect the current induced, the current will be greatest when the plane of the loop is perpendicular to the magnetic field and zero when it is parallel to the field.

Induced emf for a moving loop can be calculated using Faraday's law of magnetic induction

$$emf = -N\frac{\Delta(AB\cos\theta)}{\Delta t}$$

where A is the area of the loop, B is the strength of the magnetic field, θ is the angle the field lines make with a normal vector to the plane of the loop, and N is the number of turns in the loop. The effect of this equation is that if more magnetic field lines are cut by the loop of wire the emf will be increased, whether by moving the loop or by changing the field itself.

Lenz's law states that *the magnetic field of the induced current opposes the* ***change*** *in the applied magnetic field.*

22.2 Alternating Current, Generators, and Motors

Generators use induction to convert mechanical energy to electrical energy. The produce a continuously changing emf by rotating a coil of wire inside a uniform magnetic field. When the loop is perpendicular to the magnetic field every portion of the loop is moving parallel to the magnetic field so there is no current induced, when the loop is parallel to the magnetic field then the sides of the loop are crossing the largest number of magnetic field lines so the largest emf is then produced. The emf will vary from zero to the maximum twice every rotation of the coil and will follow a sinusoidal path, this is called **alternating current**.

The maximum potential difference is only reached during a small part of each cycle, the effective potential difference is the square root of the average of the square of the potential difference over time, known as the root-mean-square potential difference (ΔV_{rms}). This and the similar expression for current are

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\Delta V_{max}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

The rms potential difference can be used as the potential difference term and the rms current can be used as the current term in all of the electrical equations from previous chapters. Electrical meters also usually read rms values for alternating current.

Motors use an arrangement similar to that of generators to convert electrical energy into mechanical energy, the difference is that in a motor the electrical energy is converted to mechanical energy, a generator does the opposite.

22.3 Inductance

Mutual inductance involves the induction of a current in one circuit by means of a changing current in another circuit. The example seen most often is a transformer, two coils of wire wound on the same iron core, this will change the potential difference of an alternating current source. For a transformer with primary and secondary windings of N_1 and N_2 turns, an input potential difference of ΔV_1 will be changed into an output potential difference ΔV_2

$$\Delta V_2 = \frac{N_2}{N_1}\Delta V_1$$

The power produced by the output of a transformer is the same as the power consumed by the transformer, so $P_1 = P_2$ and combining the two equations eventually yields $\Delta V_1 I_1 = \Delta V_2 I_2$.

Variables and Notation

SI Prefixes

Prefix	Mult.	Abb.	Prefix	Mult.	Abb.
yocto-	10^{-24}	y	yotta-	10^{24}	Y
zepto-	10^{-21}	z	zetta-	10^{21}	Z
atto-	10^{-18}	a	exa-	10^{18}	E
femto-	10^{-15}	f	peta-	10^{15}	P
pico-	10^{-12}	p	tera-	10^{12}	T
nano-	10^{-9}	n	giga-	10^{9}	G
micro-	10^{-6}	μ	mega-	10^{6}	M
milli-	10^{-3}	m	kilo-	10^{3}	k
centi-	10^{-2}	c	hecto-	10^{2}	h
deci-	10^{-1}	d	deka-	10^{1}	da

Notation

Notation	Description
$\vec{x}$	Vector
x	Scalar, or the magnitude of $\vec{x}$
$\lvert\vec{x}\rvert$	The absolute value or magnitude of $\vec{x}$
Δx	Change in x
$\sum x$	Sum of all x
Πx	Product of all x
x_i	Initial value of x
x_f	Final value of x
$\hat{x}$	Unit vector in the direction of x
$A \rightarrow B$	A implies B
$A \propto B$	A is proportional to B
$A \gg B$	A is much larger than B

Units

Symbol	Unit	Quantity	Composition
kg	kilogram	Mass	SI base unit
m	meter	Length	SI base unit
s	second	Time	SI base unit
A	ampere	Electric current	SI base unit
cd	candela	Luminous intensity	SI base unit
K	kelvin	Temperature	SI base unit
mol	mole	Amount	SI base unit
Ω	ohm	Resistance	$\frac{\mathrm{V}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A^2}}$
C	coulomb	Charge	$\mathrm{A \cdot s}$
F	farad	Capacitance	$\frac{\mathrm{C}}{\mathrm{V}}$ or $\frac{\mathrm{s^4 \cdot A^2}}{\mathrm{m^2 \cdot kg}}$
H	henry	Inductance	$\frac{\mathrm{V \cdot s}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{A^2 \cdot s^2}}$
Hz	hertz	Frequency	$\mathrm{s^{-1}}$
J	joule	Energy	$\mathrm{N \cdot m}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$
N	newton	Force	$\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$
rad	radian	Angle	$\frac{\mathrm{m}}{\mathrm{m}}$ or 1
T	tesla	Magnetic field	$\frac{\mathrm{N}}{\mathrm{A \cdot m}}$
V	volt	Electric potential	$\frac{\mathrm{J}}{\mathrm{C}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A}}$
W	watt	Power	$\frac{\mathrm{J}}{\mathrm{s}}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}$
Wb	weber	Magnetic flux	$\mathrm{V \cdot s}$ or $\frac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot A}}$

Greek Alphabet

Name	Maj.	Min.	Name	Maj.	Min.
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π or ϖ
Epsilon	E	ϵ or ε	Rho	P	ρ or ϱ
Zeta	Z	ζ	Sigma	Σ	σ or ς
Eta	H	η	Tau	T	τ
Theta	Θ	θ or ϑ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ or φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Variables

Variable	Description	Units
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ	Angular position	rad
θ_c	Critical angle	^{o} (degrees)
θ_i	Incident angle	^{o} (degrees)
θ_r	Refracted angle	^{o} (degrees)
θ'	Reflected angle	^{o} (degrees)
$\Delta\theta$	Angular displacement	rad
τ	Torque	$\text{N} \cdot \text{m}$
ω	Angular speed	$\frac{\text{rad}}{\text{s}}$
μ	Coefficient of friction	(unitless)
μ_k	Coefficient of kinetic friction	(unitless)
μ_s	Coefficient of static friction	(unitless)
$\vec{a}$	Acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_c$	Centripetal acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_g$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_t$	Tangential acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_x$	Acceleration in the x direction	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_y$	Acceleration in the y direction	$\frac{\text{m}}{\text{s}^2}$
A	Area	m^2
B	Magnetic field strength	T
C	Capacitance	F
$\vec{d}$	Displacement	m
$d\sin\theta$	lever arm	m
$\vec{d}_x$ or Δx	Displacement in the x direction	m
$\vec{d}_y$ or Δy	Displacement in the y direction	m
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
f	Focal length	m
$\vec{F}$	Force	N
$\vec{F}_c$	Centripetal force	N
$\vec{F}_{electric}$	Electrical force	N
$\vec{F}_g$	Gravitational force	N
$\vec{F}_k$	Kinetic frictional force	N
$\vec{F}_{magnetic}$	Magnetic force	N
$\vec{F}_n$	Normal force	N
$\vec{F}_s$	Static frictional force	N
$\vec{F}\Delta t$	Impulse	$\text{N} \cdot \text{s}$ or $\frac{\text{kg} \cdot \text{m}}{\text{s}}$
$\vec{g}$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$

Variable	Description	Units
h	Object height	m
h'	Image height	m
I	Current	A
I	Moment of inertia	$\text{kg} \cdot \text{m}^2$
KE or K	Kinetic energy	J
KE_{rot}	Rotational kinetic energy	J
L	Angular Momentum	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}}$
L	Self-inductance	H
m	Mass	kg
M	Magnification	(unitless)
M	Mutual inductance	H
MA	Mechanical Advantage	(unitless)
ME	Mechanical Energy	J
n	Index of refraction	(unitless)
p	Object distance	m
$\vec{p}$	Momentum	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$
P	Power	W
PE or U	Potential Energy	J
$PE_{elastic}$	Elastic potential energy	J
$PE_{electric}$	Electrical potential energy	J
PE_g	Gravitational potential energy	J
q	Image distance	m
q or Q	Charge	C
Q	Heat, Entropy	J
R	Radius of curvature	m
R	Resistance	Ω
s	Arc length	m
t	Time	s
Δt	Time interval	s
$\vec{v}$	Velocity	$\frac{\text{m}}{\text{s}}$
v_t	Tangential speed	$\frac{\text{m}}{\text{s}}$
$\vec{v}_x$	Velocity in the x direction	$\frac{\text{m}}{\text{s}}$
$\vec{v}_y$	Velocity in the y direction	$\frac{\text{m}}{\text{s}}$
V	Electric potential	V
ΔV	Electric potential difference	V
V	Volume	m^3
W	Work	J
x or y	Position	m

Constants

Symbol	Name	Established Value	Value Used
ϵ_0	Permittivity of a vacuum	$8.854\ 187\ 817 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}$	$8.85 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}$
ϕ	Golden ratio	1.618 033 988 749 894 848 20	
π	Archimedes' constant	3.141 592 653 589 793 238 46	
g, a_g	Gravitational acceleration constant	$9.79171\ \frac{\mathrm{m}}{\mathrm{s}^2}$ (varies by location)	$9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$
c	Speed of light in a vacuum	$2.997\ 924\ 58 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$ (exact)	$3.00 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$
e	Natural logarithmic base	2.718 281 828 459 045 235 36	
e^-	Elementary charge	$1.602\ 177\ 33 \times 10^{19}$ C	1.60×10^{19} C
G	Gravitational constant	$6.672\ 59 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg}^2}$	$6.67 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg}^2}$
k_C	Coulomb's constant	$8.987\ 551\ 788 \times 10^{9}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2}$	$8.99 \times 10^{9}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2}$
N_A	Avogadro's constant	$6.022\ 141\ 5 \times 10^{23}\ \mathrm{mol}^{-1}$	

Astronomical Data

Symbol	Object	Mean Radius	Mass	Mean Orbit Radius	Orbital Period
☽	Moon	1.74×10^{6} m	7.36×10^{22} kg	3.84×10^{8} m	2.36×10^{6} s
☉	Sun	6.96×10^{8} m	1.99×10^{30} kg	—	—
☿	Mercury	2.43×10^{6} m	3.18×10^{23} kg	5.79×10^{10} m	7.60×10^{6} s
♀	Venus	6.06×10^{6} m	4.88×10^{24} kg	1.08×10^{11} m	1.94×10^{7} s
♁	Earth	6.37×10^{6} m	5.98×10^{24} kg	1.496×10^{11} m	3.156×10^{7} s
♂	Mars	3.37×10^{6} m	6.42×10^{23} kg	2.28×10^{11} m	5.94×10^{7} s
	Ceres[1]	4.71×10^{5} m	9.5×10^{20} kg	4.14×10^{11} m	1.45×10^{8} s
♃	Jupiter	6.99×10^{7} m	1.90×10^{27} kg	7.78×10^{11} m	3.74×10^{8} s
♄	Saturn	5.85×10^{7} m	5.68×10^{26} kg	1.43×10^{12} m	9.35×10^{8} s
♅	Uranus	2.33×10^{7} m	8.68×10^{25} kg	2.87×10^{12} m	2.64×10^{9} s
♆	Neptune	2.21×10^{7} m	1.03×10^{26} kg	4.50×10^{12} m	5.22×10^{9} s
♇	Pluto[1]	1.15×10^{6} m	1.31×10^{22} kg	5.91×10^{12} m	7.82×10^{9} s
	Eris[21]	2.4×10^{6} m	1.5×10^{22} kg	1.01×10^{13} m	1.75×10^{10} s

[1] Ceres, Pluto, and Eris are classified as "Dwarf Planets" by the IAU
[2] Eris was formerly known as 2003 UB_{313}

Mathematics Review for Physics

This is a summary of the most important parts of mathematics as we will use them in a physics class. There are numerous parts that are completely omitted, others are greatly abridged. Do not assume that this is a complete coverage of any of these topics.

Algebra

Fundamental properties of algebra

$a+b=b+a$	Commutative law for addition
$(a+b)+c=a+(b+c)$	Associative law for addition
$a+0=0+a=a$	Identity law for addition
$a+(-a)=(-a)+a=0$	Inverse law for addition
$ab=ba$	Commutative law for multiplication
$(ab)c=a(bc)$	Associative law for multiplication
$(a)(1)=(1)(a)=a$	Identity law for multiplication
$a\frac{1}{a}=\frac{1}{a}a=1$	Inverse law for multiplication
$a(b+c)=ab+ac$	Distributive law

Exponents

$$(ab)^n = a^n b^n \qquad (a/b)^n = a^n/b^n$$
$$a^n a^m = a^{n+m} \qquad 0^n = 0$$
$$a^n/a^m = a^{n-m} \qquad a^0 = 1$$
$$(a^n)^m = a^{(mn)} \qquad 0^0 = 1 \text{ (by definition)}$$

Logarithms

$$x = a^y \rightarrow y = \log_a x$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^n) = n\log_a x$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a} = (\log_b x)(\log_a b)$$

Binomial Expansions

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a^i b^{n-i}$$

Quadratic formula

For equations of the form $ax^2+bx+c=0$ the solutions are:

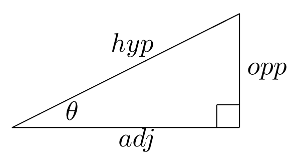
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Geometry

Shape	Area	Volume
Triangle	$A=\frac{1}{2}bh$	—
Rectangle	$A=lw$	—
Circle	$A=\pi r^2$	—
Rectangular prism	$A=2(lw+lh+hw)$	$V=lwh$
Sphere	$A=4\pi r^2$	$V=\frac{4}{3}\pi r^3$
Cylinder	$A=2\pi rh+2\pi r^2$	$V=\pi r^2 h$
Cone	$A=\pi r\sqrt{r^2+h^2}+\pi r^2$	$V=\frac{1}{3}\pi r^2 h$

Trigonometry

In physics only a small subset of what is covered in a trigonometry class is likely to be used, in particular sine, cosine, and tangent are useful, as are their inverse functions. As a reminder, the relationships between those functions and the sides of a right triangle are summarized as follows:



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

The inverse functions are only defined over a limited range. The $\tan^{-1} x$ function will yield a value in the range $-90° < \theta < 90°$, $\sin^{-1} x$ will be in $-90° \leq \theta \leq 90°$, and $\cos^{-1} x$ will yield one in $0° \leq \theta \leq 180°$. Care must be taken to ensure that the result given by a calculator is in the correct quadrant, if it is not then an appropriate correction must be made.

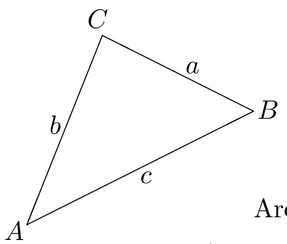
Degrees	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

$$\sin^2\theta + \cos^2\theta = 1 \qquad 2\sin\theta\cos\theta = \sin(2\theta)$$

Trigonometric functions in terms of each other

$\sin\theta =$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$
$\cos\theta =$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$
$\tan\theta =$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$
$\sec\theta =$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$
$\cot\theta =$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$
$\sin\theta =$	$\frac{1}{\csc\theta}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\frac{1}{\sqrt{\csc^2\theta-1}}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\csc\theta$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\sqrt{\csc^2\theta-1}$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Law of sines, law of cosines, area of a triangle



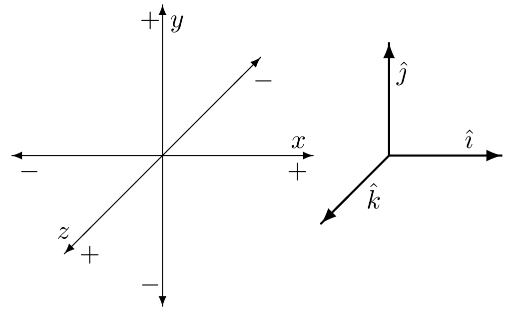
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\text{let } s = \tfrac{1}{2}(a+b+c)$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area} = \tfrac{1}{2}bc\sin A = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}ab\sin C$$

Vectors

A **vector** is a quantity with both magnitude and direction, such as displacement or velocity. Your textbook indicates a vector in bold-face type as **V** and in class we have been using $\vec{V}$. Both notations are equivalent.

A **scalar** is a quantity with only magnitude. This can either be a quantity that is directionless such as time or mass, or it can be the magnitude of a vector quantity such as speed or distance traveled. Your textbook indicates a scalar in italic type as V, in class we have not done anything to distinguish a scalar quantity. The magnitude of $\vec{V}$ is written as V or $|\vec{V}|$.

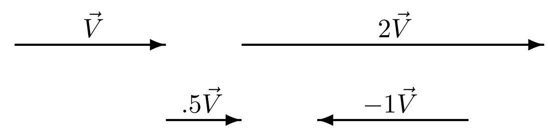
A **unit vector** is a vector with magnitude 1 (a dimensionless constant) pointing in some significant direction. A unit vector pointing in the direction of the vector $\vec{V}$ is indicated as $\hat{V}$ and would commonly be called V-hat. Any vector can be normalized into a unit vector by dividing it by its magnitude, giving $\hat{V} = \frac{\vec{V}}{V}$. Three special unit vectors, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are introduced with chapter 3. They point in the directions of the positive x, y, and z axes, respectively (as shown below).



Vectors can be added to other vectors of the same dimension (i.e. a velocity vector can be added to another velocity vector, but not to a force vector). The sum of all vectors to be added is called the **resultant** and is equivalent to all of the vectors combined.

Multiplying Vectors

Any vector can be multiplied by any scalar, this has the effect of changing the magnitude of the vector but not its direction (with the exception that multiplying a vector by a negative scalar will reverse the direction of the vector). As an example, multiplying a vector $\vec{V}$ by several scalars would give:



In addition to scalar multiplication there are also two ways to multiply vectors by other vectors. They will not be directly used in class but being familiar with them may help to understand how some physics equations are derived. The first, the **dot product** of vectors $\vec{V}_1$ and $\vec{V}_2$, represented as $\vec{V}_1 \cdot \vec{V}_2$ measures the tendency of the two vectors to point in the same direction. If the angle between the two vectors is θ the dot product yields a scalar value as

$$\vec{V}_1 \cdot \vec{V}_2 = V_1 V_2 \cos\theta$$

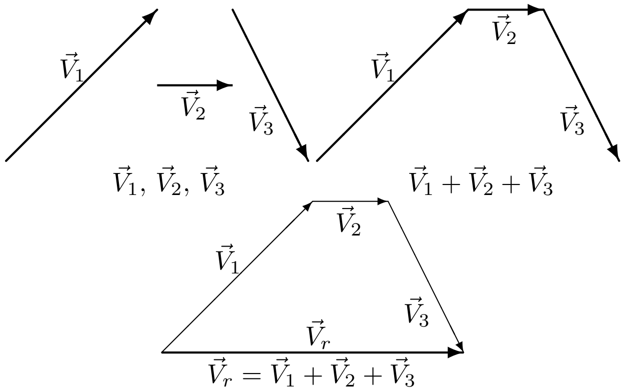
The second method of multiplying two vectors, the **cross product**, (represented as $\vec{V}_1 \times \vec{V}_2$) measures the tendency of vectors to be perpendicular to each other. It yields a third vector perpendicular to the two original vectors with magnitude

$$|\vec{V}_1 \times \vec{V}_2| = V_1 V_2 \sin\theta$$

The direction of the cross product is perpendicular to the two vectors being crossed and is found with the right-hand rule — point the fingers of your right hand in the direction of the first vector, curl them toward the second vector, and the cross product will be in the direction of your thumb.

Adding Vectors Graphically

The sum of any number of vectors can be found by drawing them head-to-tail to scale and in proper orientation then drawing the resultant vector from the tail of the first vector to the point of the last one. If the vectors were drawn accurately then the magnitude and direction of the resultant can be measured with a ruler and protractor. In the example below the vectors $\vec{V}_1$, $\vec{V}_2$, and $\vec{V}_3$ are added to yield $\vec{V}_r$

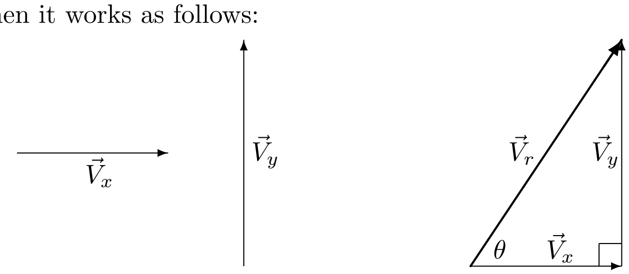


Adding Parallel Vectors

Any number of parallel vectors can be directly added by adding their magnitudes if one direction is chosen as positive and vectors in the opposite direction are assigned a negative magnitude for the purposes of adding them. The sum of the magnitudes will be the magnitude of the resultant vector in the positive direction, if the sum is negative then the resultant will point in the negative direction.

Adding Perpendicular Vectors

Perpendicular vectors can be added by drawing them as a right triangle and then finding the magnitude and direction of the hypotenuse (the resultant) through trigonometry and the Pythagorean theorem. If $\vec{V}_r = \vec{V}_x + \vec{V}_y$ and $\vec{V}_x \perp \vec{V}_y$ then it works as follows:



Since the two vectors to be added and the resultant form a right triangle with the resultant as the hypotenuse the Pythagorean theorem applies giving

$$V_r = |\vec{V}_r| = \sqrt{V_x^2 + V_y^2}$$

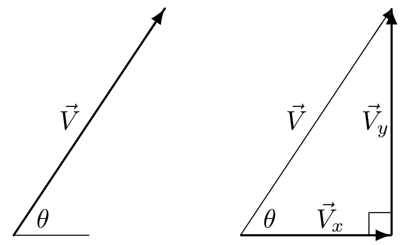
The angle θ can be found by taking the inverse tangent of the ratio between the magnitudes of the vertical and horizontal vectors, thus

$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

As was mentioned above, care must be taken to ensure that the angle given by the calculator is in the appropriate quadrant for the problem, this can be checked by looking at the diagram drawn to solve the problem and verifying that the answer points in the direction expected, if not then make an appropriate correction.

Resolving a Vector Into Components

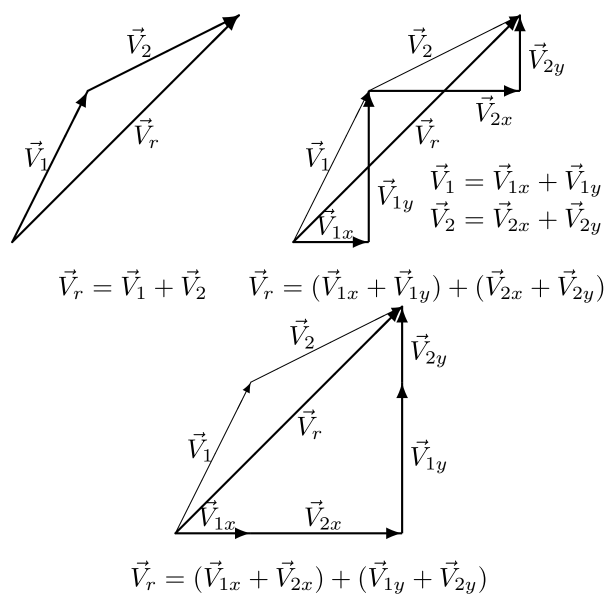
Just two perpendicular vectors can be added to find a single resultant, any single vector $\vec{V}$ can be resolved into two perpendicular **component vectors** $\vec{V}_x$ and $\vec{V}_y$ so that $\vec{V} = \vec{V}_x + \vec{V}_y$.



As the vector and its components can be drawn as a right triangle the ratios of the sides can be found with trigonometry. Since $\sin\theta = \frac{V_y}{V}$ and $\cos\theta = \frac{V_x}{V}$ it follows that $V_x = V\cos\theta$ and $V_y = V\sin\theta$ or in a vector form, $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$. (This is actually an application of the dot product, $\vec{V}_x = (\vec{V}\cdot\hat{\imath})\hat{\imath}$ and $\vec{V}_y = (\vec{V}\cdot\hat{\jmath})\hat{\jmath}$, but it is not necessary to know that for this class)

Adding Any Two Vectors Algebraically

Only vectors with the same direction can be directly added, so if vectors pointing in multiple directions must be added they must first be broken down into their components, then the components are added and resolved into a single resultant vector — if in two dimensions $\vec{V}_r = \vec{V}_1 + \vec{V}_2$ then



Once the sums of the component vectors in each direction have been found the resultant can be found from them just as an other perpendicular vectors may be added. Since from the last figure $\vec{V}_r = (\vec{V}_{1x}+\vec{V}_{2x})+(\vec{V}_{1y}+\vec{V}_{2y})$ and it was previously established that $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$ it follows that

$$\vec{V}_r = (V_1\cos\theta_1 + V_2\cos\theta_2)\,\hat{\imath} + (V_1\sin\theta_1 + V_2\sin\theta_2)\,\hat{\jmath}$$

and

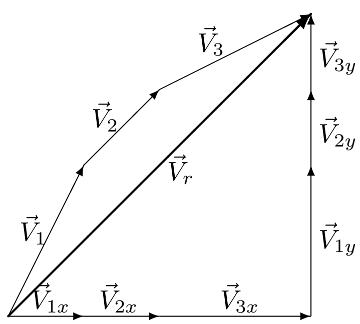
$$\begin{aligned} V_r &= \sqrt{(V_{1x}+V_{2x})^2 + (V_{1y}+V_{2y})^2} \\ &= \sqrt{(V_1\cos\theta_1 + V_2\cos\theta_2)^2 + (V_1\sin\theta_1 + V_2\sin\theta_2)^2} \end{aligned}$$

with the direction of the resultant vector $\vec{V}_r$, θ_r, being found with

$$\theta_r = \tan^{-1}\frac{V_{1y}+V_{2y}}{V_{1x}+V_{2x}} = \tan^{-1}\frac{V_1\sin\theta_1 + V_2\sin\theta_2}{V_1\cos\theta_1 + V_2\cos\theta_2}$$

Adding Any Number of Vectors Algebraically

For a total of n vectors $\vec{V}_i$ being added with magnitudes V_i and directions θ_i the magnitude and direction are:



$$\vec{V}_r = \left(\sum_{i=1}^{n} V_i\cos\theta_i\right)\hat{\imath} + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)\hat{\jmath}$$

$$V_r = \sqrt{\left(\sum_{i=1}^{n} V_i\cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)^2}$$

$$\theta_r = \tan^{-1}\frac{\sum_{i=1}^{n} V_i\sin\theta_i}{\sum_{i=1}^{n} V_i\cos\theta_i}$$

(The figure shows only three vectors but this method will work with any number of them so long as proper care is taken to ensure that all angles are measured the same way and that the resultant direction is in the proper quadrant.)

Calculus

Although this course is based on algebra and not calculus, it is sometimes useful to know some of the properties of derivatives and integrals. If you have not yet learned calculus it is safe to skip this section.

Derivatives

The **derivative** of a function $f(t)$ with respect to t is a function equal to the slope of a graph of $f(t)$ vs. t at every point, assuming that slope exists. There are several ways to indicate that derivative, including:

$$\frac{\mathrm{d}}{\mathrm{d}t}f(t)$$
$$f'(t)$$
$$\dot{f}(t)$$

The first one from that list is unambiguous as to the independent variable, the others assume that there is only one variable, or in the case of the third one that the derivative is taken with respect to time. Some common derivatives are:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}t^n &= nt^{n-1} \\ \frac{\mathrm{d}}{\mathrm{d}t}\sin t &= \cos t \\ \frac{\mathrm{d}}{\mathrm{d}t}\cos t &= -\sin t \\ \frac{\mathrm{d}}{\mathrm{d}t}e^t &= e^t \end{aligned}$$

If the function is a compound function then there are a few useful rules to find its derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}cf(t) = c\frac{\mathrm{d}}{\mathrm{d}t}f(t) \qquad c \text{ constant for all } t$$
$$\frac{\mathrm{d}}{\mathrm{d}t}[f(t)+g(t)] = \frac{\mathrm{d}}{\mathrm{d}t}f(t) + \frac{\mathrm{d}}{\mathrm{d}t}g(t)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}f(u) = \frac{\mathrm{d}}{\mathrm{d}t}u\frac{\mathrm{d}}{\mathrm{d}u}f(u)$$

Integrals

The **integral**, or **antiderivative** of a function $f(t)$ with respect to t is a function equal to the the area under a graph of $f(t)$ vs. t at every point plus a constant, assuming that $f(t)$ is continuous. Integrals can be either indefinite or definite. An indefinite integral is indicated as:

$$\int f(t)\,\mathrm{d}t$$

while a definite integral would be indicated as

$$\int_a^b f(t)\,\mathrm{d}t$$

to show that it is evaluated from $t = a$ to $t = b$, as in:

$$\int_a^b f(t)\,\mathrm{d}t = \int f(t)\,\mathrm{d}t\Big|_{t=a}^{t=b} = \int f(t)\,\mathrm{d}t\Big|_{t=b} - \int f(t)\,\mathrm{d}t\Big|_{t=a}$$

Some common integrals are:

$$\int t^n\,\mathrm{d}t = \frac{t^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int t^{-1}\,\mathrm{d}t = \ln t + C$$
$$\int \sin t\,\mathrm{d}t = -\cos t + C$$
$$\int \cos t\,\mathrm{d}t = \sin t + C$$
$$\int e^{t}\,\mathrm{d}t = e^{t} + C$$

There are rules to reduce integrals of some compound functions to simpler forms (there is no general rule to reduce the integral of the product of two functions):

$$\int cf(t)\,\mathrm{d}t = c\int f(t)\,\mathrm{d}t \qquad c \text{ constant for all } t$$
$$\int [f(t)+g(t)]\,\mathrm{d}t = \int f(t)\,\mathrm{d}t + \int g(t)\,\mathrm{d}t$$

Series Expansions

Some functions are difficult to work with in their normal forms, but once converted to their series expansion can be manipulated easily. Some common series expansions are:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i+1}}{(2i+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i}}{(2i)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{i=0}^{\infty}\frac{x^i}{(i)!}$$

Physics Using Calculus

In our treatment of mechanics the majority of the equations that have been covered in the class have been either approximations or special cases where the acceleration or force applied have been held constant. This is required because the class is based on algebra and to do otherwise would require the use of calculus.

None of what is presented here is a required part of the class, it is here to show how to handle cases outside the usual approximations and simplifications. Feel free to skip this section, none of it will appear as a required part of any assignment or test in this class.

Notation

Because the letter 'd' is used as a part of the notation of calculus the variable 'r' is often used to represent the position vector of the object being studied. (Other authors use 's' for the spatial position or generalize 'x' to two or more dimensions.) In a vector form that would become $\vec{r}$ to give both the magnitude and direction of the position. It is assumed that the position, velocity, acceleration, etc. can be expressed as a function of time as $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ but the dependency on time is usually implied and not shown explicitly.

Translational Motion

Many situations will require an acceleration that is not constant. Everything learned about translational motion so far used the simplification that the acceleration remained constant, with calculus we can let the acceleration be any function of time.

Since velocity is the slope of a graph of position vs. time at any point, and acceleration is the slope of velocity vs. time these can be expressed mathematically as derivatives:

$$\vec{v} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}$$

$$\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}$$

The reverse of this relationship is that the displacement of an object is equal to the area under a velocity vs. time graph and velocity is equal to the area under an acceleration vs. time graph. Mathematically this can be expressed as integrals:

$$\vec{v} = \int \vec{a}\,\mathrm{d}t$$

$$\vec{r} = \int \vec{v}\,\mathrm{d}t = \iint \vec{a}\,\mathrm{d}t^2$$

Using these relationships we can derive the main equations of motion that were introduced by starting with the integral of a constant velocity $\vec{a}(t) = \vec{a}$ as

$$\int_{t_i}^{t_f} \vec{a}\,\mathrm{d}t = \vec{a}\Delta t + C$$

The constant of integration, C, can be shown to be the initial velocity, $\vec{v}_i$, so the entire expression for the final velocity becomes

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

or

$$\vec{v}_f(t) = \vec{v}_i + \vec{a}t$$

Similarly, integrating that expression with respect to time gives an expression for position:

$$\int_{t_i}^{t_f} (\vec{v}_i + \vec{a}t)\,\mathrm{d}t = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2 + C$$

Once again the constant of integration, C, can be shown to be the initial position, $\vec{r}_i$, yielding an expression for position vs. time

$$\vec{r}_f(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

While calculus can be used to derive the algebraic forms of the equations the reverse is not true. Algebra can handle a subset of physics and only at the expense of needing to remember separate equations for various special cases. With calculus the few definitions shown above will suffice to predict any linear motion.

Work and Power

The work done on an object is the integral of dot product of the force applied with the path the object takes, integrated as a contour integral over the path.

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The power developed is the time rate of change of the work done, or the derivative of the work with respect to time.

$$P = \frac{\mathrm{d}}{\mathrm{d}t}W$$

Momentum

Newton's second law of motion changes from the familiar $\vec{F} = m\vec{a}$ to the statement that the force applied to an object is the time derivative of the object's momentum.

$$\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{p}$$

Similarly, the impulse delivered to an object is the integral of the force applied in the direction parallel to the motion with respect to time.

$$\Delta\vec{p} = \int \vec{F}\,\mathrm{d}t$$

Rotational Motion

The equations for rotational motion are very similar to those for translational motion with appropriate variable substitutions. First, the angular velocity is the time derivative of the angular position.

$$\vec{\omega} = \frac{\mathrm{d}\vec{\theta}}{\mathrm{d}t}$$

The angular acceleration is the time derivative of the angular velocity or the second time derivative of the angular position.

$$\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\theta}}{\mathrm{d}t^2}$$

The angular velocity is also the integral of the angular acceleration with respect to time.

$$\vec{\omega} = \int \vec{\alpha}\,\mathrm{d}t$$

The angular position is the integral of the angular velocity with respect to time or the second integral of the angular acceleration with respect to time.

$$\vec{\theta} = \int \vec{\omega}\,\mathrm{d}t = \iint \vec{\alpha}\,\mathrm{d}t^2$$

The torque exerted on an object is the cross product of the radius vector from the axis of rotation to the point of action with the force applied.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The moment of inertia of any object is the integral of the square of the distance from the axis of rotation to each element of the mass over the entire mass of the object.

$$I = \int r^2\,\mathrm{d}m$$

The torque applied to an object is the time derivative of the object's angular momentum.

$$\vec{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

The integral of the torque with respect to time is the angular momentum of the object.

$$\vec{L} = \int \vec{\tau}\,\mathrm{d}t$$

Data Analysis

Comparative Measures

Percent Error

When experiments are conducted where there is a calculated result (or other known value) against which the observed values will be compared the percent error between the observed and calculated values can be found with

$$\textit{percent error} = \left| \frac{\textit{observed value} - \textit{accepted value}}{\textit{accepted value}} \right| \times 100\%$$

Percent Difference

When experiments involve a comparison between two experimentally determined values where neither is regarded as correct then rather than the percent error the percent difference can be calculated. Instead of dividing by the accepted value the difference is divided by the mean of the values being compared. For any two values, a and b, the percent difference is

$$\textit{percent difference} = \left| \frac{a-b}{\frac{1}{2}(a+b)} \right| \times 100\%$$

Dimensional Analysis

Unit Conversions

When converting units you need to be careful to ensure that conversions are done properly and not accidentally reversed (or worse). The easiest way to do this is to be careful to always keep the units associated with each number and at each step always multiply by something equal to 1.

As an example, if you have a distance of 78.25 km and want to convert it to meters the immediate reaction is to say that you can multiply by 1000, or is that divide by 1000. Knowing that 1 km = 1000 m it is obvious then that $\frac{1\text{ km}}{1000\text{ m}} = 1 = \frac{1000\text{ m}}{1\text{ km}}$. Since both of them are equal to 1 they can be multiplied by what you wish to convert without changing the quantity itself, it will only change the units in which it is shown. So, to convert 78.25 km to meters you would multiply by $\frac{1000\text{ m}}{1\text{ km}}$:

$$(78.25\ \cancel{\text{km}})\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right) = 78\ 250\text{ m}$$

As you can see, the km in the original quantity and in the denominator cancel leaving meters. If you were to reverse the conversion accidentally you would get this:

$$(78.25\text{ km})\left(\frac{1\text{ km}}{1000\text{ m}}\right) = 0.07825\ \frac{\text{km}^2}{\text{m}}$$

Instead of canceling the km and leaving the measurement in meters the result is in $\frac{\text{km}^2}{\text{m}}$, a meaningless quantity. The nonsensical result is an indication that the conversion was reversed and should be redone. If the conversion was done without the units then it could be reversed without realizing that it happened.

Units on Constants

Consider a mass vibrating on the end of a spring. The equation relating the mass to the frequency on graph might look something like

$$y = \frac{3.658}{x^2} - 1.62$$

This isn't what we're looking for, a first pass would be to change the x and y variables into f and m for frequency and mass, this would then give

$$m = \frac{3.658}{f^2} - 1.62$$

This is progress but there's still a long way to go. Mass is measured in kg and frequency is measured in s^{-1}, since these variables represent the entire measurement (including the units) and not just the numeric portion they do not need to be labeled, but the constants in the equation do need to have the correct units applied.

To find the units it helps to first re-write the equation using just the units, letting some variable such as u (or u_1, u_2, u_3, etc.) stand for the unknown units. The example equation would then become

$$\text{kg} = \frac{u_1}{(\text{s}^{-1})^2} + u_2$$

Since any quantities being added or subtracted must have the same units the equation can be split into two equations

$$\text{kg} = \frac{u_1}{(\text{s}^{-1})^2} \quad \text{and} \quad \text{kg} = u_2$$

The next step is to solve for u_1 and u_2, in this case $u_2 = \text{kg}$ is immediately obvious, u_1 will take a bit more effort. A first simplification yields

$$\text{kg} = \frac{u_1}{\text{s}^{-2}}$$

then multiplying both sides by s^{-2} gives

$$(\text{s}^{-2})(\text{kg}) = \left(\frac{u_1}{\text{s}^{-2}}\right)(\text{s}^{-2})$$

which finally simplifies and solves to

$$u_1 = \text{kg} \cdot \text{s}^{-2}$$

Inserting the units into the original equation finally yields

$$m = \frac{3.658\text{ kg} \cdot \text{s}^{-2}}{f^2} - 1.62\text{ kg}$$

which is the final equation with all of the units in place. Checking by choosing a frequency and carrying out the calculations in the equation will show that it is dimensionally consistent and yields a result in the units for mass (kg) as expected.

Final Exam Description

- 95 Multiple Choice Items (22 problems, 73 conceptual questions)
- Calculators will **not** be permitted
- Exam items are from the following categories:
  - **Chapter 1 — The Science of Physics**
    units of measurement
    significant figures
  - **Chapter 2 — Motion in One Dimension**
    graphs
    equations of motion
    acceleration
  - **Chapter 3 — Two-Dimensional Motion and Vectors**
    vectors and scalars
    vector addition
    components
    projectiles
    relative motion
  - **Chapter 4 — Forces and the Laws of Motion**
    causes of motion
    inertia
    balanced forces
    accelerated motion
    friction
  - **Chapter 5 — Work and Energy**
    work
    types of energy
    conservation of energy
    power
  - **Chapter 6 — Momentum and Collisions**
    momentum and impulse
    conservation of momentum
    types of collisions
  - **Chapter 7 — Rotational Motion and the Law of Gravity**
    rotational motion equations
    centripetal force and acceleration
    causes of centripetal force
    gravitational force equation
  - **Chapter 8 — Rotational Equilibrium and Dynamics**
    torque
    center of gravity
    equilibrium
  - **Chapter 12 — Vibrations and Waves**
    simle harmonic motion
    wave properties
    interference
    standing waves
  - **Chapter 13 — Sound**
    types of waves
    wave speed
    interference
    Doppler shift
    resonance
  - **Chapter 14 — Light and Reflection**
    characteristics of light
    flat mirrors
    curved mirrors
    polarization
  - **Chapter 15 — Refraction**
    index of refraction
    Snell's law
    lenses
  - **Chapter 16 — Interference and Diffraction**
    interference
    diffraction
  - **Chapter 17 — Electric Forces and Fields**
    electric charge
    methods of producing net charge
    conductors and insulators
    electric forces
  - **Chapter 18 — Electrical Energy and Capacitance**
    electric potential energy
    potential difference
  - **Chapter 19 — Current and Resistance**
    current
    resistance
    Ohm's law
    power
  - **Chapter 20 — Circuits and Circuit Elements**
    series and parallel resistors
    resistor combinations
  - **Chapter 21 — Magnetism**
    causes of magnetism
    magnetic fields
    current and magnetism
    magnetic forces
  - **Chapter 22 — Induction and Alternating Current**
    induced current
    Lenz's law
    motors and generators
    transformers

Final Exam Equation Sheet

This is a slightly reduced copy of the front and back of the equation sheet that you will receive with the final exam.

Honors Physics – 2nd Semester Final
Constants and Equations

Kinematics
Linear

$$d = v_{av}t$$

$$d = \tfrac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$d = v_i\Delta t + \tfrac{1}{2}a\left(\Delta t\right)^2$$

$$v_f = v_i + a\Delta t$$

$$v_f^2 = v_i^2 + 2ad$$

$$d_x = v_x\Delta t$$

$$d_y = v_y\Delta t + \tfrac{1}{2}a_y\left(\Delta t\right)^2$$

$$v_y = v\sin\theta$$

$$v_x = v\cos\theta$$

Kinematics
Rotational

$$\theta = \frac{d}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a}{r}$$

$$\theta = \omega_{av}\Delta t$$

$$\theta = \omega_i\Delta t + \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

$$\theta = \tfrac{1}{2}\left(\omega_i + \omega_f\right)\Delta t$$

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

Dynamics
Linear

$$\sum \vec{F} = m\vec{a}$$

$$F_g = mg$$

$$F_{\perp} = w\cos\theta$$

$$F_{\parallel} = w\sin\theta$$

$$f = \mu F_N$$

$$\mu = \tan\theta$$

$$F_{spring} = -kx$$

$$F_{grav} = G\frac{m_1 m_2}{r^2}$$

Dynamics
Circular

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F_c = m\frac{v^2}{r} = m\frac{4\pi^2 r}{T^2}$$

$$T = 1/f$$

Dynamics
Rotational

$$\sum \tau = I\alpha$$

$$I_{\text{point mass}} = mr^2$$

$$\sum \tau_{counterclockwise} = \sum \tau_{clockwise}$$

Work, Power, Energy
Linear

$$W = Fd\cos\theta$$

$$PE_g = mgh$$

$$PE_{elastic} = \tfrac{1}{2}kx^2$$

$$KE = \tfrac{1}{2}mv^2$$

$$\Delta PE + \Delta KE + W = 0$$

$$P = \frac{W}{t} = Fv$$

Work, Power, Energy
Rotational

$$W = \tau\theta$$

$$KE = \tfrac{1}{2}I\omega^2$$

$$P = \tau\omega$$

Momentum
Linear

$$P = mv$$

$$m_1 v_{1i} + m_2 v_{2f} = m_1 v_{1f} + m_2 v_{2f}$$

$$F\Delta t = m\Delta v$$

Rotational

$$L = I\omega$$

Constants

$$g = 10.\ \mathrm{m/s^2}$$

$$G = 6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}$$

$$c = 3.00\times10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$K = 9.00\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$$

Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$x = A\cos(\omega t)$$

$$v = -\omega A\sin(\omega t)$$

$$a = -\omega^2 A\cos(\omega t)$$

$$a = -\frac{k}{m}x$$

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T = \frac{1}{f}$$

Waves and Sound

$$v = f\lambda$$

$$v = 20.1\sqrt{T_k} = 331\,{}^{m}\!/\!{}_{s}\sqrt{1 + \frac{T_c}{273}}$$

$$v = 331\,{}^{m}\!/\!{}_{s} + .6T_c$$

$$f = f_s\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

$$\#\,Beats = \left|f_1 - f_2\right|$$

Light

$$\theta_i = \theta_r$$

$$n \equiv \frac{v_{light\ \text{in vacuum}}}{v_{light\ \text{in medium}}}$$

$$n = \frac{c}{v} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$n_i\sin\theta_i = n_r\sin\theta_r$$

$$\frac{v_i}{v_r} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\sin\theta_{i_c} = {}^{1}\!/\!{}_{n}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{s_i}{s_o} = \frac{d_i}{d_o}$$

Electrostatics

$$F = k\frac{q_1 q_2}{r^2}$$

$$E = \frac{|F|}{|q_o|} = k\frac{|q|}{r^2}$$

$$\Delta V \equiv V_B - V_A = \frac{\Delta PE_e}{q}$$

$$V = k\frac{q}{r}$$

$$W = -q\Delta V = -q\left(V_B - V_A\right)$$

$$PE_e = qEd$$

Currents and Circuits

$$I \equiv \frac{\Delta q}{\Delta t}$$

$$V = IR$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

$$C \equiv \frac{q}{V}$$

Series Circuits

$$I_t = I_1 = I_2 = I_3 = \ldots$$

$$V_t = V_1 + V_2 + V_3 + \ldots$$

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Parallel Circuits

$$I_t = I_1 + I_2 + I_3 + \ldots$$

$$V_t = V_1 = V_2 = V_3 = \ldots$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

Magnetism

$$F = qvB\sin\theta$$

$$F = I\ell B\sin\theta$$

$$V = El = Blv$$

Transformers

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$

$$V_p I_p = V_s I_s$$

Note that some equations may be slightly different than the ones we have used in class, the sheet that you receive will have these forms of the equations, it will be up to you to know how to use them or to know other forms that you are able to use.

Mixed Review Exercises

Chapter 1

HOLT PHYSICS

Mixed Review

The Science of Physics

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	m
10^{-3}	milli-	m
10^{-2}	centi-	c

Power	Prefix	Abbreviation
10^{-1}	deci-	d
10^{1}	deka-	da
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

1. Convert the following measurements to the units specified.
   a. 2.5 days to seconds ______
   b. 35 km to millimeters ______
   c. 43 cm to kilometers ______
   d. 22 mg to kilograms ______
   e. 671 kg to micrograms ______
   f. 8.76×10^{7} mW to gigawatts ______
   g. 1.753×10^{-13} s to picoseconds ______
2. According to the rules given in Chapter 1 of your textbook, how many significant figures are there in the following measurements?
   a. 0.0845 kg ______
   b. 37.00 h ______
   c. 8 630 000.000 mi ______
   d. 0.000 000 0217 g ______
   e. 750 in. ______
   f. 0.5003 s ______

Chapter 1 HOLT PHYSICS Mixed Review *continued*

3. **Without calculating the result,** find the number of significant figures in the following products and quotients.
   a. 0.005032×4.0009 ______
   b. $0.0080750 \div 10.037$ ______
   c. $(3.52 \times 10^{-11}) \times (7.823 \times 10^{11})$ ______
4. Calculate $a + b$, $a - b$, $a \times b$, and $a \div b$ with the correct number of significant figures using the following numbers.
   a. $a = 0.005\ 078$; $b = 1.0003$

      $a + b =$ ______ $a - b =$ ______

      $a \times b =$ ______ $a \div b =$ ______
   b. $a = 4.231\ 19 \times 10^{7}$; $b = 3.654 \times 10^{6}$

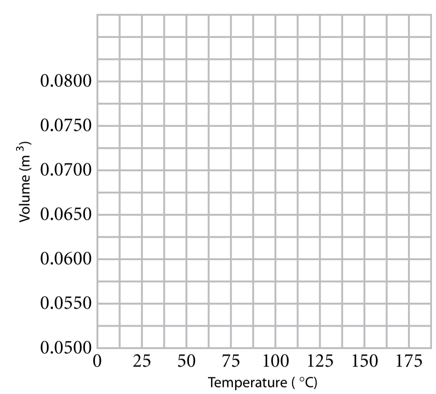
      $a + b =$ ______ $a - b =$ ______

      $a \times b =$ ______ $a \div b =$ ______
5. Calculate the area of a carpet 6.35 m long and 2.50 m wide. Express your answer with the correct number of significant figures.

   ______
6. The table below contains measurements of the temperature and volume of an air balloon as it heats up.

   In the grid at right, sketch a graph that best describes these data.

Temperature (°C)	Volume (m^3)
2	0.0502
27	0.0553
52	0.0598
77	0.0646
102	0.0704
127	0.0748
152	0.0796



Chapter 2

HOLT PHYSICS

Mixed Review

Motion in One Dimension

1. During a relay race along a straight road, the first runner on a three-person team runs d_1 with a constant velocity v_1. The runner then hands off the baton to the second runner, who runs d_2 with a constant velocity v_2. The baton is then passed to the third runner, who completes the race by traveling d_3 with a constant velocity v_3.

   a. In terms of d and v, find the time it takes for each runner to complete a segment of the race.

   Runner 1 __________ Runner 2 __________ Runner 3 __________

   b. What is the total distance of the race course?

   c. What is the total time it takes the team to complete the race?

2. The equations below include the equations for straight-line motion. For each of the following problems, indicate which equation or equations you would use to solve the problem, but do not actually perform the calculations.

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t \qquad \Delta x = \tfrac{1}{2}(v_f)\Delta t$$
$$\Delta x = v_i(\Delta t) + \tfrac{1}{2}a(\Delta t)^2 \qquad \Delta x = \tfrac{1}{2}a(\Delta t)^2$$
$$v_f = v_i + a(\Delta t) \qquad v_f = a(\Delta t)$$
$$v_f^2 = v_i^2 + 2a\Delta x \qquad v_f^2 = 2a\Delta x$$

   a. During takeoff, a plane accelerates at 4 m/s^2 and takes 40 s to reach takeoff speed. What is the velocity of the plane at takeoff?

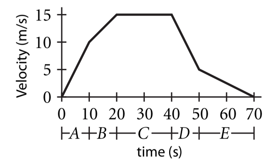
   b. A car with an initial speed of 31.4 km/h accelerates at a uniform rate of 1.2 m/s^2 for 1.3 s. What is the final speed and displacement of the car during this time?

Chapter 2

HOLT PHYSICS

Mixed Review *continued*

3. Below is the velocity-time graph of an object moving along a straight path. Use the information in the graph to fill in the table below.



For each of the lettered intervals below, indicate the motion of the object (whether it is speeding up, slowing down, or at rest), the direction of the velocity (+, –, or 0), and the direction of the acceleration (+, –, or 0).

Time interval	Motion	v	a
A			
B			
C			
D			
E			

4. A ball is thrown upward with an initial velocity of 9.8 m/s from the top of a building.

   a. Fill in the table below showing the ball's position, velocity, and acceleration at the end of each of the first 4 s of motion.

Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
1			
2			
3			
4			

   b. In which second does the ball reach the top of its flight?

   c. In which second does the ball reach the level of the roof, on the way down?

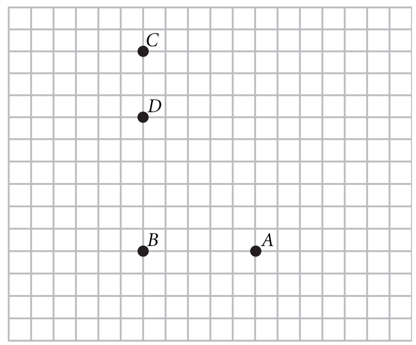
Chapter 3

HOLT PHYSICS

Mixed Review

Two-Dimensional Motion and Vectors

1. The diagram below indicates three positions to which a woman travels. She starts at position *A*, travels 3.0 km to the west to point *B*, then 6.0 km to the north to point *C*. She then backtracks, and travels 2.0 km to the south to point *D*.



   a. In the space provided, diagram the displacement vectors for each segment of the woman's trip.

   b. What is the total displacement of the woman from her initial position, *A*, to her final position, *D*?

   c. What is the total distance traveled by the woman from her initial position, *A*, to her final position, *D*?

2. Two projectiles are launched from the ground, and both reach the same vertical height. However, projectile B travels twice the horizontal distance as projectile A before hitting the ground.

   a. How large is the vertical component of the initial velocity of projectile B compared with the vertical component of the initial velocity of projectile A?

   b. How large is the horizontal component of the initial velocity of projectile B compared with the horizontal component of the initial velocity of projectile A?

   c. Suppose projectile A is launched at an angle of 45° to the horizontal. What is the ratio, v_B/v_A, of the speed of projectile B, v_B, compared with the speed of projectile A, v_A?

Chapter 3

HOLT PHYSICS

Mixed Review *continued*

3. A passenger at an airport steps onto a moving sidewalk that is 100.0 m long and is moving at a speed of 1.5 m/s. The passenger then starts walking at a speed of 1.0 m/s in the same direction as the sidewalk is moving. What is the passenger's velocity relative to the following observers?

   a. A person standing stationary alongside to the moving sidewalk.

   b. A person standing stationary *on* the moving sidewalk.

   c. A person walking alongside the sidewalk with a speed of 2.0 m/s and in a direction opposite the motion of the sidewalk.

   d. A person riding in a cart alongside the sidewalk with a speed of 5.0 m/s and in the same direction in which the sidewalk is moving.

   e. A person riding in a cart with a speed of 4.0 m/s and in a direction perpendicular to the direction in which the sidewalk is moving.

4. Use the information given in item 3 to answer the following questions:

   a. How long does it take for the passenger walking on the sidewalk to get from one end of the sidewalk to the other end?

   b. How much time does the passenger save by taking the moving sidewalk instead of walking alongside it?

Chapter 4

HOLT PHYSICS

Mixed Review

Forces and the Laws of Motion

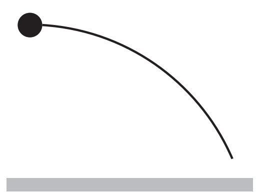
1. A crate rests on the horizontal bed of a pickup truck. For each situation described below, indicate the motion of the crate relative to the ground, the motion of the crate relative to the truck, and whether the crate will hit the front wall of the truck bed, the back wall, or neither. Disregard friction.

   a. Starting at rest, the truck *accelerates* to the right.

   b. The crate is at rest relative to the truck while the truck moves to the right with a constant velocity.

   c. The truck in item b slows down.

2. A ball with a mass of m is thrown through the air, as shown in the figure.



   a. What is the gravitational force exerted on the ball by Earth?

   b. What is the force exerted on Earth by the ball?

   c. If the surrounding air exerts a force on the ball that resists its motion, is the *total* force on the ball the same as the force calculated in part a?

   d. If the surrounding air exerts a force on the ball that resists its motion, is the *gravitational* force on the ball the same as the force calculated in part a?

3. Two blocks of masses m_1 and m_2, respectively, are placed in contact with each other on a smooth, horizontal surface. A constant horizontal force F to the right is applied to m_1. Answer the following questions in terms of F, m_1, and m_2.

   a. What is the acceleration of the two blocks?

   b. What are the horizontal forces acting on m_2?

   c. What are the horizontal forces acting on m_1?

   d. What is the magnitude of the contact force between the two blocks?

4. Assume you have the same situation as described in item 3, only this time there is a frictional force, F_k, between the blocks and the surface. Answer the following questions in terms of F, F_k, m_1, and m_2.

   a. What is the acceleration of the two blocks?

   b. What are the horizontal forces acting on m_2?

   c. What are the horizontal forces acting on m_1?

   d. What is the magnitude of the contact force between the two blocks?

Chapter 5 HOLT PHYSICS

Mixed Review

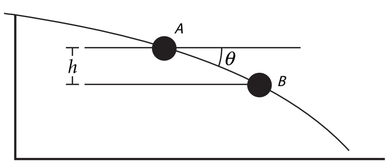
Work and Energy

1. A ball has a mass of 3 kg. What is the work done on this ball by the gravitational force exerted by Earth if the ball moves 2 m along each of the following directions?

   a. downward (along the force) ______________

   b. upward (opposite the force) ______________

2. A stone with a mass of m is thrown off a building. As the stone passes point A, it has a speed of v_A at an angle of q to the horizontal. The stone then travels a vertical distance h to point B, where it has a speed v_B.



   a. What is the work done on the stone by the gravitational force due to Earth while the stone moves from A to B?

   b. What is the change in the kinetic energy of the stone as it moves from A to B?

   c. What is the speed v_B of the stone in terms of v_A, g, and h?

   d. Does the change in the stone's speed between A and B depend on the mass of the stone?

   e. Does the change in the stone's speed between A and B depend on the angle q?

Chapter 5 HOLT PHYSICS

Mixed Review *continued*

3. An empty coffee mug with a mass of 0.40 kg gets knocked off a tabletop 0.75 m above the floor onto the seat of a chair 0.45 m above the floor. Assume that the gravitational potential energy, PE_g, is measured using the floor as the zero energy level.

   a. What is the initial gravitational potential energy associated with the mug's position on the table?

   b. What is the final gravitational potential energy associated with the mug's position on the chair seat?

   c. What was the work done by the gravitational force as it fell from the table to the chair?

   d. Suppose that zero level for the energy was taken to be the ceiling of the room rather than the floor . Would the answers to items a to c be the same or different?

4. A carton of shoes with a mass of m slides with an initial speed of v_i m/s down a ramp inclined at an angle of 23° to the horizontal. The carton's initial height is h_i, and its final height is h_f, and it travels a distance of d down the ramp. There is a frictional force, F_k, between the ramp and the carton.

   a. What is the initial mechanical energy, ME_i, of the carton? (Hint: Apply the law of conservation of energy.)

   b. If mis the coefficient of friction between the ramp and the carton, what is F_k?

   c. Express the final speed, v_f, of the carton in terms of v_i, g, d, and m

Chapter 6

HOLT PHYSICS

Mixed Review

Momentum and Collisions

1. A pitcher throws a softball toward home plate. The ball may be hit, sending it back toward the pitcher, or it may be caught, bringing it to a stop in the catcher's mitt.

   a. Compare the change in momentum of the ball in these two cases.

   b. Discuss the magnitude of the impulse on the ball in these two cases.

   c. In the space below, draw a vector diagram for each case, showing the initial momentum of the ball, the impulse exerted on the ball, and the resulting final momentum of the ball.

2. a. Using Newton's third law, explain why the impulse on one object in a collision is equal in magnitude but opposite in direction to the impulse on the second object.

   b. Extend your discussion of impulse and Newton's third law to the case of a bowling ball striking a set of 10 bowling pins.

3. Starting with the conservation of total momentum, $\mathbf{p_f} = \mathbf{p_i}$, show that the final velocity for two objects in an inelastic collision is

$$\mathbf{v_f} = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v_{1,i}} + \left(\frac{m_2}{m_1 + m_2}\right)\mathbf{v_{2,i}}.$$

4. Two moving billiard balls, each with a mass of M, undergo an elastic collision. Immediately before the collision, ball A is moving east at 2 m/s and ball B is moving east at 4 m/s.

   a. In terms of M, what is the total momentum (magnitude and direction) immediately before the collision?

   b. The final momentum, $M(\mathbf{v_{A,f}} + \mathbf{v_{B,f}})$, must equal the initial momentum. If the final velocity of ball A increases to 4 m/s east because of the collision, what is the final momentum of ball B?

   c. For each ball, compare the final momentum of the ball to the initial momentum of the other ball. These results are typical of head-on elastic collisions. What generalization about head-on elastic collisions can you make?

Chapter 7

HOLT PHYSICS

Mixed Review

Rotational Motion and the Law of Gravity

1. Complete the following table.

	s (m)	r (m)	Δq (rad)	Δt (s)	w (rad/s)	v_t (m/s)	a_c (m/s^2)
a.	4.5		1.5	0.50			
b.		0.50	8.5		8.5		
c.	3.2	0.20			58		
d.	1250		2.0	17			
e.	3750	750				86	

2. Describe the force that maintains circular motion in the following cases.

a. A car exits a freeway and moves around a circular ramp to reach the street below.

____

____

b. The moon orbits Earth.

____

____

c. During gym class, a student hits a tether ball on a string.

____

____

3. Determine the change in gravitational force under the following changes.

a. one of the masses is doubled ______________________________

b. both masses are doubled ______________________________

c. the distance between masses is doubled ______________________________

d. the distance between masses is halved ______________________________

e. the distance between masses is tripled ______________________________

Chapter 7

HOLT PHYSICS

Mixed Review *continued*

4. Some plans for a future space station make use of rotational force to simulate gravity. In order to be effective, the centripetal acceleration at the outer rim of the station should equal about 1 g, or 9.81 m/s^2. However, humans can withstand a difference of only 1/100 g between their head and feet before they become disoriented. Assume the average human height is 2.0 m, and calculate the minimum radius for a safe, effective station. (Hint: The ratio of the centripetal acceleration of astronaut's feet to the centripetal acceleration of the astronaut's head must be at least 99/100.)

5. As an elevator begins to descend, you feel momentarily lighter. As the elevator stops, you feel momentarily heavier. Sketch the situation, and explain the sensations using the forces in your sketch.

6. Two cars start on opposite sides of a circular track. One car has a speed of 0.015 rad/s; the other car has a speed of 0.012 rad/s. If the cars start p radians apart, calculate the time it takes for the faster car to catch up with the slower car.

Chapter 8

HOLT PHYSICS
Mixed Review

Rotational Equilibrium and Dynamics

1. a. On some doors, the doorknob is in the center of the door. What would a physicist say about the practicality of this arrangement? Why would physicists design doors with knobs farther from the hinge?

   b. How much more force would be required to open the door from the center rather than from the edge?

2. Figure skaters commonly change the shape of their body in order to achieve spins on the ice. Explain the effects on each of the following quantities when a figure skater pulls in his or her arms.

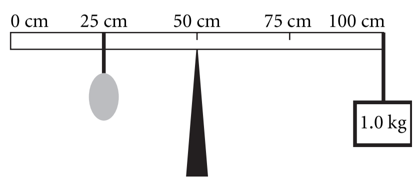
   a. moment of inertia

   b. angular momentum

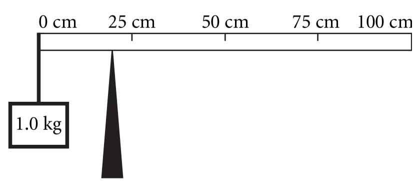
   c. angular speed

3. For the following items, assume the objects shown are in rotational equilibrium.

   a. What is the mass of the sphere to the right?



   b. What is the mass of the portion of the meter-stick to the left of the pivot? (Hint: 20% of the mass of the meterstick is on the left. How much must be on the right?)



4. A force of 25 N is applied to the end of a uniform rod that is 0.50 m long and has a mass of 0.75 kg.

   a. Find the torque, moment of inertia, and angular acceleration if the rod is allowed to pivot around its center of mass.

   b. Find the torque, moment of inertia, and angular acceleration if the rod is allowed to pivot around the end, away from the applied force.

5. A satellite in orbit around Earth is initially at a constant angular speed of 7.27×10^{-5} rad/s. The mass of the satellite is 45 kg, and it has an orbital radius of 4.23×10^{7} m.

   a. Find the moment of inertia of the satellite in orbit around Earth.

   b. Find the angular momentum of the satellite.

   c. Find the rotational kinetic energy of the satellite around Earth.

   d. Find the tangential speed of the satellite.

   e. Find the translational kinetic energy of the satellite.

6. A series of two simple machines is used to lift a 13300 N car to a height of 3.0 m. Both machines have an efficiency of 0.90 (90 percent). Machine A moves the car, and the output of machine B is the input to machine A.

   a. How much work is done on the car?

   b. How much work must be done on machine A in order to achieve the amount of work done on the car?

   c. How much work must be done on machine B in order to achieve the amount of work from machine A?

   d. What is the overall efficiency of this process?

Chapter 12

HOLT PHYSICS

Mixed Review

Vibrations and Waves

1. A pendulum with a mass of 0.100 kg was released. The string made a 7.0° angle with the vertical. The bob of the pendulum returns to its lowest point every 0.10 s.

   a. What is its period? What is its frequency?

   b. The pendulum is replaced by one with a mass of 0.300 kg and set to swing with a 15° angle. Do the following quantities increase, decrease, or remain the same?

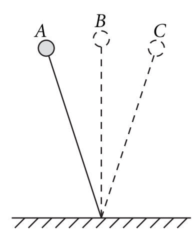
   period ______

   frequency ______

   total energy ______

   speed at the lowest point ______

2. A narrow, flat steel rod is anchored at its lower end, with a 0.500 kg ball welded to the top end. A force of 6.00 N is required to hold the ball 10.0 cm away from its central position.



   If this arrangement is modeled as an oscillating horizontal mass-spring system, vibrating with a simple harmonic motion, find

   a. the force constant, *k*, of the spring.

   b. the period and frequency of the oscillations.

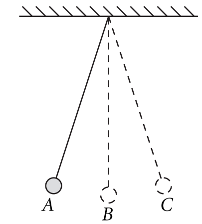
3. Find the acceleration due to gravity at a place where a simple pendulum 0.150 m long completes 1.00×10^2 oscillations in 3.00×10^2 seconds. Could this place be on Earth?

Chapter 12

HOLT PHYSICS

Mixed Review *continued*

4. Consider the first two cycles of a pendulum swinging from position *A* with a period of 2.00 s.



   a. At which times is the bob found at positions *A*, *B*, and *C* during the first two cycles?

   b. At which times and locations is gravitational potential energy at a maximum? At which times is kinetic energy at a maximum?

   c. At which times and locations is the velocity at a maximum? the restoring force? the acceleration?

5. The frequency of a pressure wave is 1.00×10^2 Hz. Its wavelength is 3.00 m. Find the speed of wave propagation.

6. A pressure wave of 0.50 m wavelength propagates through a 3.00 m long coil spring at a speed of 2.00 m/s. How long does it take for the wave to travel from one end of the coil to the other? How many wavelengths fit in the coil?

Chapter 13 HOLT PHYSICS

Mixed Review

Sound

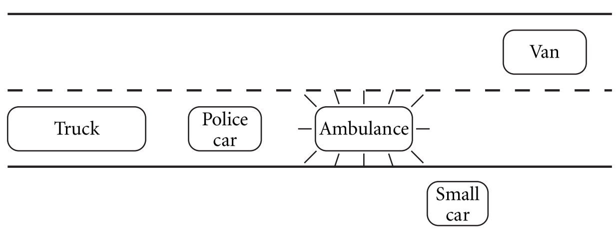
1. The speed of sound increases with temperature. It is 331 m/s in air at 0°C and 343 m/s in air at 20°C. A glass pipe vibrates with a frequency of 151 Hz.

   a. What is the wavelength of the sound produced by the column of air in the pipe on a cold day (0°C) and on a warmer day (20°C)?

   b. How does air temperature affect the wavelength of the sound produced by the pipe?

2. The driver of an ambulance turns on its siren as the ambulance heads east at 30 mph. A police car is following the ambulance at 30 mph. A truck behind the police car is moving at 20 mph. A van is traveling west in the opposite lane at 20 mph. A small car is stopped at the side of the road. The vehicles are positioned as shown.

   a. On the diagram, sketch and label arrows to indicate the velocity of each vehicle.



   b. Rank the sounds perceived by the passengers in each of the vehicles in order of decreasing frequency.

3. A 330 Hz tuning fork is vibrating after being struck. It is placed on a table near but not directly touching other objects, including other tuning forks. Eventually one glass and one other tuning fork start vibrating. Explain why this happens.

4. The first harmonic in a pipe closed at one end is 487 Hz.

   a. Find the next two harmonic frequencies that will occur in this pipe.

   b. What are the corresponding wavelengths of the first three harmonics? (Hint: assume the speed of sound is 345 m/s.)

   c. What is the length of this pipe?

   d. Repeat this exercise for a pipe open at both ends.

5. A piano tuner uses a 440 Hz tuning fork to tune a string that is currently vibrating at 445 Hz.

   a. How many beats per second does he hear?

   b. What other frequency could produce the same sound effect? Explain why.

Chapter 14

HOLT PHYSICS

Mixed Review

Light and Reflection

1. Proxima Centauri, the nearest star in our galaxy, is 4.30 light-years away. What is its distance in meters?

2. Radio signals emitted from and received by an airplane have a frequency of 3.00×10^{12} Hz and travel at the speed of light.
   a. How long is the delay in each message going from the control tower to a jet flying at 1.00×10^{4} m of altitude?
   b. What is the wavelength of these signals?

3. A laser beam is sent to the moon from Earth. The reflected beam is received on Earth after 2.56 seconds. What is the distance from Earth to the moon?

4. The background radiation in the universe (believed to come from the Big Bang) includes microwaves with wavelengths of 0.100 cm. What is the frequency of this radiation?

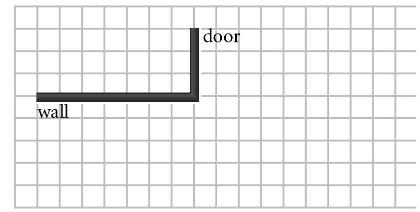
5. List five objects that reflect light diffusely. List three objects that reflect light specularly for the most part.

   Diffuse reflection ______

   Specular reflection ______

Chapter 14 HOLT PHYSICS Mixed Review *continued*

6. A mirror door is located next to a large wall mirror. The door is closed to create a 90° angle with the wall. You stand 2.00 m from the door and 1.00 m from the wall.



   a. On the diagram at right, sketch a top-view diagram of the situation at scale. Label the object (yourself) *A*.
   b. Locate your first image in the mirror on the door. Label it *B*. Locate *B*'s image in the mirror on the wall. Label it *C*.
   c. Locate your first image in the mirror on the wall and its image in the mirror on the door. Label them *D* and *E*.
   d. Where will the next images of the images be located?

7. An object located 36.0 cm from a concave mirror produces a real image located 12.0 cm from the mirror.
   a. Find the focal length of this mirror
   b. Find the location, type, and size of the image formed by a 6.00 cm tall object located 30.0 cm, 24.0 cm, 18.0 cm, 12.0 cm, and 6.00 cm in front of the mirror.

8. The concave mirror in the problem above is replaced by a convex one with the same curvature. Find the location of the images produced when the object is located 30.0 cm, 24.0 cm, 18.0 cm, 12.0 cm, and 6.00 cm in front of the mirror.

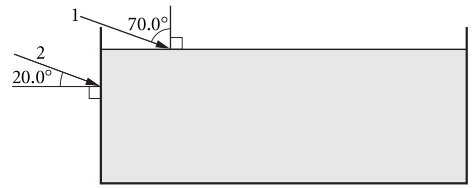
Chapter 15

HOLT PHYSICS

Mixed Review

Refraction

1. Two parallel rays enter an aquarium as shown. Ray 1 forms a 70.0° angle with the normal to the surface. Ray 2 forms a 20.0° angle with the normal to the wall. (Hint: the index of refraction for water is 1.33.)

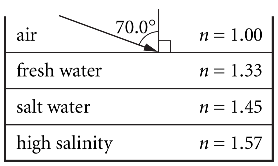


   a. Calculate the angle of refraction of each ray.

   b. Trace the path of each light ray inside the water.

   c. Are the refracted rays inside the water still parallel? Will they intersect in the water?

2. A large beaker contains layers of water of increasing salinity, separated by a thin plastic plate. The lowest layer has the highest salinity and refractive index, as shown in the diagram. A ray of light strikes the surface of fresh water at the top, at a 70.0° angle from the normal.



   a. Find the angles of refraction and the angles of incidence at each boundary.

   b. There is a flat mirror at the bottom of the container. Trace the path of one light ray coming from the air to the bottom of the beaker and back.

Chapter 15

HOLT PHYSICS

Mixed Review *continued*

3. An object located 36.0 cm from a thin converging lens has a real image located 12.0 cm from the lens.

   a. Find the focal point of this lens.

   b. Find the location, type, and size of the image formed by a 6.00 cm tall object located 30.0 cm, 24.0 cm, 18.0 cm, 12.0 cm, and 6.00 cm in front of the lens.

4. The converging lens in item 3 is replaced by a diverging lens. Now the image of the first object is located 12.0 cm in front of the lens. Find the focal distance of the diverging lens and the location of the images produced when the object is placed at the distances described in item 3b.

5. A bug placed 1.00 cm under a magnifying glass appears exactly six times larger.

   a. Where is the bug's image located?

   b. What is the focal point of the lens in the magnifying glass?

Chapter 16 HOLT PHYSICS

Mixed Review

Interference and Diffraction

1. The second-order bright fringes of interference are observed at an 8.53° angle in a double-slit experiment with light of 5.00×10^2 nm wavelength.

   a. Determine the slits' separation.

   b. Find the angle of the tenth-order bright fringe.

   c. In this experiment, the screen is 2.00 m wide. Its distance from the source is 1.00 m. Could the tenth-order fringe be observed? Why or why not?

2. Diffraction of white light with a single slit produces bright lines of different colors.

   a. Which wavelengths are more diffracted by the same slit size?

   b. In the space below, sketch a diagram showing the location of red, green and blue lines of the first and second order. Describe the sequence in which the colors appear, beginning with the color closest to the center.

   c. What is the color of the central image?

3. You have three diffraction gratings. Grating A has 2.0×10^5 lines per meter. Grating B has 9.0×10^6 lines per meter. Grating C has 3.0×10^7 lines per meter.

   a. What is the slit distance of each grating?

   b. Which gratings can diffract the following:

      - visible light of 500 nm wavelength
      - X rays of 5.00 nm wavelength
      - infrared light of 5000 nm wavelength

4. You drop pebbles into the water on a rocky beach. When the waves you made reach the rocks, new waves appear to start in the spaces between the rocks.

   a. Are these waves coherent?

   b. How is this like a double slit illuminated by a single light source?

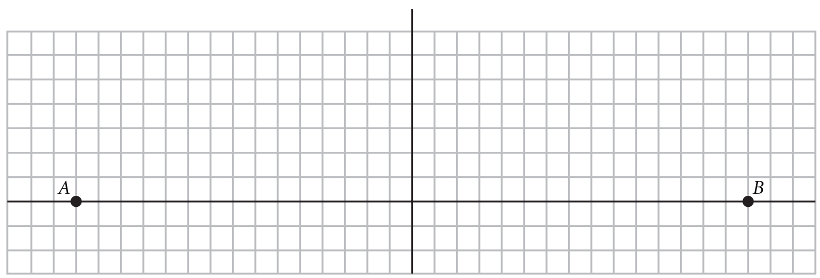
Chapter 17 HOLT PHYSICS

Mixed Review

Electric Forces and Fields

Use $k_C = 8.99 \times 10^9$ N•m^2/C^2.

1. Two spheres, *A* and *B*, are placed 0.60 m apart, as shown. Sphere *A* has +3.00 μC excess charge. Sphere *B* has +5.00 μC excess charge.



   a. How many electrons are missing on sphere *A*? on sphere *B*?

   b. How do the forces of *B* on *A* and *A* on *B* compare? Does the greater charge exert a greater force?

2. A third spherical charge, *C*, of +2.00 μC, is placed on the line connecting spheres *A* and *B*. Find the resultant force exerted by *A* and *B* on *C* when *C* is placed in the following locations.

   a. 0.20 m to the left of *A*

   b. 0.20 m to the right of *A* between *A* and *B*

   c. exactly in the middle between *A* and *B*

Chapter 17 HOLT PHYSICS

Mixed Review *continued*

3. Alpha particles are made of two protons and two neutrons.
$m_p = 1.673 \times 10^{-27}$ kg; $m_n = 1.675 \times 10^{-27}$ kg; $q_e = 1.60 \times 10^{-19}$ C

   a. Find the electric force acting on an alpha particle in a horizontal electric field of 6.00×10^2 N/C.

   b. What is the acceleration of this alpha particle?

   c. How does this acceleration compare with gravity? Describe the particle's trajectory. Will it be close to horizontal? to vertical free fall?

4. A 2.00 μC point charge of mass 5.00 g is suspended on a string and placed in a horizontal electric field. The mass is in equilibrium when the string forms a 17.3° angle with the vertical.

   a. In the space below, sketch a free-body diagram of the problem. Show the vertical and horizontal components of the tension force in the string.

   b. Find the electric force on the charge in this field.

   c. Find the strength of the electric field.

5. How many electrons are there in 1.00 C? How many electrons are there in 1.00 μC?

Chapter 19

HOLT PHYSICS

Mixed Review

Current and Resistance

1. A 60.0 cm metal wire draws 0.185 A from a 36.0 V battery. Will the current increase or decrease when the following changes are performed? Explain whether the change is due to a change in resistance, a change in potential difference, or other reasons.

   a. The wire is cut into four pieces, and only one segment is used.

   b. The wire is bent to form an *M* shape.

   c. The wire is heated to 500°C.

   d. The 36.0 V battery is replaced by a 24.0 V battery.

2. A 25 Ω resistance heater is connected to a potential difference of 120 V for 5.00 h.

   a. How much current does the heater draw?

   b. How much electric charge travels through the heating element during this time?

   c. What is the power consumption of the heater?

   d. Use the power and time to calculate how much energy was consumed.

3. The label on a three-way light bulb package specifies 100 W, 150 W, 250 W, 120 V.

   a. How much current does the light bulb draw in each of the three ways? (Assume three significant figures in each of these measurements.)

   b. What is the bulb's resistance in each way?

   c. Compare the cost of using the light bulb for 100.0 h in each way. (Assume that the price is 7.00 ¢/kWh.)

4. An electric hot plate draws 6.00 A of current when its resistance is 24.0 Ω.

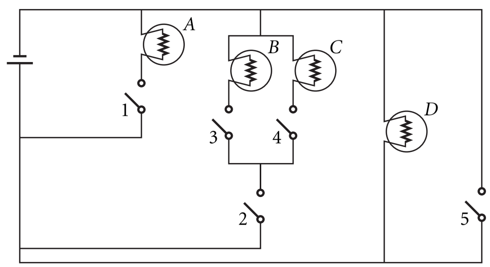
   a. What is the voltage across the hot plate's heating element?

   b. How much power does it consume?

   c. For what length of time should it be kept on in order to supply 9×10^4 J to a coffeepot? (Assume that all electrical energy is transferred to the coffeepot by heat.)

Chapter 20 HOLT PHYSICS
Mixed Review
Circuits and Circuit Elements

1. Consider the circuit shown below.



a. Do any of the bulbs have a complete circuit when all the switches are open? Which one(s)? ______

b. Do any of the switches cause a short circuit when closed? Which one(s)? ______

c. Which switches should be kept open, and which should be closed for the following to occur?

- only bulbs *A* and *B* are off ______
- only bulbs *A* and *C* are off ______
- only bulbs *B* and *C* are off ______

2. A light bulb of unknown resistance is connected in series with a 9.0 Ω resistor to a 12.0 V battery. The current in the bulb is 0.80 A.

a. In the space below, sketch a schematic diagram of the circuit.

b. Find the equivalent resistance of the circuit.

______

c. Find the resistance of the light bulb.

______

Chapter 20 HOLT PHYSICS
Mixed Review *continued*

3. A light bulb of unknown resistance is connected in parallel to a 48.0 Ω resistor and to a 12.0 V battery. The current through the battery is 2.50 A.

a. In the space below, sketch a schematic diagram of the circuit.

b. Find the potential difference across the resistor and across the bulb.

______

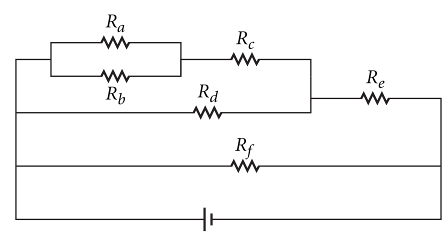
c. Find the current in the resistor and in the bulb.

______

d. Find the resistance of the light bulb.

______

4. In the circuit below, find the equivalent resistance for the following situations.



a. $R_a = R_b = R_c = R_d = R_e = R_f = 10.0\ \Omega$

______

b. $R_a = 10.0\ \Omega$; $R_b = 20.0\ \Omega$; $R_c = 30.0\ \Omega$; $R_d = 40.0\ \Omega$; $R_e = 50.0\ \Omega$; $R_f = 60.0\ \Omega$

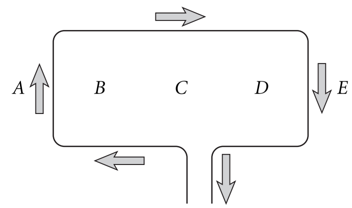
______

Chapter 21 HOLT PHYSICS

Mixed Review

Magnetism

1. A wire frame carries an electric current in the direction shown. Consider the magnetic field contributed by each segment of the frame at points *A, B, C, D,* and *E.*



   a. Use the convention symbols (×, •, and →) to represent the direction of magnetic fields created at point *A* by the vertical segments of the frame. Do they have the same direction? the same strength?

   b. Repeat for the horizontal segments.

   c. Answer items a and b for points *B, C, D,* and *E,* and fill in the table below.

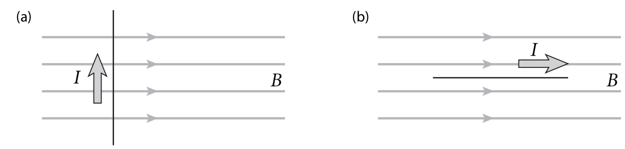
	leftmost	rightmost	upper	lower
B				
C				
D				
E				

   d. Do the contributions of each segment to the magnetic field cancel out at the center? Explain.

   e. Is the magnetic field resulting from the combined effects of the four sides of the frame stronger inside or outside the frame?

Chapter 21 HOLT PHYSICS Mixed Review *continued*

2. A 2.0 m long conducting wire has a current of 5.0 in a uniform magnetic field of 0.43 T. The field is parallel to the *x*-axis.



   a. What is the force on the wire when it is vertical, parallel to the *y*-axis as shown **a**?

   b. What is the force on the wire when it is horizontal, parallel to the *x*-axis as shown in **b**?

3. The wire in item 2 is bent to form a 0.50 m × 0.50 m square carrying the same 5.0 A current, with the positive charges moving clockwise in the frame. The frame is in the same magnetic field ($B = 0.43$ T).

   a. Sketch a diagram of the situation. Use arrows to indicate the direction of the current in each segment of the frame.

   b. Find the forces acting on each side of the frame. Specify their magnitude and direction.

   c. Do the forces on the frame cancel each other? Will the frame be able to move? Will it be able to rotate? Explain.

Chapter 22

HOLT PHYSICS

Mixed Review

Induction and Alternating Current

1. Which of the following actions will induce an emf in a conductor?
   a. Move a magnet near the conductor.
   b. Move the conductor near a magnet.
   c. Rotate the conductor in a magnetic field.
   d. Change the magnetic field strength.
   e. all of the above

2. A circular loop (10 turns) with a radius of 29 cm is in a magnetic field that oscillates uniformly between 0.95 T and 0.45 T with a period of 1.00 s.
   a. How much time is required for the field to change from 0.95 T to 0.45 T?
   b. What is the cross-sectional area of one turn of the loop?
   c. Assuming that the loop is perpendicular to the magnetic field, what is the induced emf in the loop?

3. Electric generators convert mechanical energy into electrical energy.
   a. What are the requirements for generating emf?
   b. The mechanical energy input is usually rotational motion. What are two possible sources of rotational motion?

4. A 250-turn generator with circular loops of radius 15 cm rotates at 60.0 rpm in a magnetic field with a strength of 1.00 T.
   a. What is the angular speed of the loops?
   b. What is the area of one loop?
   c. What is the maximum emf?
   d. What is the rms emf?

5. An electric motor is sometimes called a generator in reverse. Explain your understanding of this statement.

6. Consider a two-coil transformer joined by a common iron core.
   a. If the current in the primary side is increased, what happens to the magnetic field in the core?
   b. What effect does the answer to item 6a have on the secondary coil?
   c. Fully explain the effect of reducing the current to the primary side of a transformer.

Mixed Review Answers

Chapter 1 Mixed Review

1. a. 2.2×10^5 s
   b. 3.5×10^7 mm
   c. 4.3×10^{-4} km
   d. 2.2×10^{-5} kg
   e. 6.71×10^{11} mg
   f. 8.76×10^{-5} GW
   g. 1.753×10^{-1} ps
2. a. 3
   b. 4
   c. 10
   d. 3
   e. 2
   f. 4
3. a. 4
   b. 5
   c. 3
4. a. 1.0054; –0.9952; 5.080×10^{-3}; 5.076×10^{-3}
   b. 4.597×10^7; 3.866×10^7; 1.546×10^{14}; 11.58
5. 15.9 m^2
6. The graph should be a straight line.

Chapter 2 Mixed Review

1. a. $t_1 = d_1/v_1$; $t_2 = d_2/v_2$; $t_3 = d_3/v_3$
   b. total distance $= d_1 + d_2 + d_3$
   c. total time $= t_1 + t_2 + t_3$
2. a. $v_f = a(\Delta t)$
   b. $v_f = v_i + a(\Delta t)$; $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ or $\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2$

3.

Time interval	Type of motion	v(m/s)	a(m/s^2)
A	speeding up	+	+
B	speeding up	+	+
C	constant velocity	+	0
D	slowing down	+	–
E	slowing down	+	–

4. a.

Time (s)	Position (m)	v(m/s)	a(m/s^2)
1	4.9	0	–9.81
2	0	–9.8	–9.81
3	–14.7	–19.6	–9.81
4	–39.2	–29.4	–9.81

   b. 1 s
   c. 2 s

Chapter 3 Mixed Review

1. a. The diagram should indicate the relative distances and directions for each segment of the path.
   b. 5.0 km, slightly north of northwest
   c. 11.0 km
2. a. The same
   b. Twice as large
   c. 1.58
3. a. 2.5 m/s, in the direction of the sidewalk's motion
   b. 1.0 m/s, in the direction of the sidewalk's motion
   c. 4.5 m/s, in the direction of the sidewalk's motion
   d. 2.5 m/s, in the direction opposite to the sidewalk's motion
   e. 4.7 m/s, q = 32°
4. a. 4.0×10^1 seconds
   b. 6.0×10^1 seconds

Chapter 4 Mixed Review

1. a. at rest, moves to the left, hits back wall
   b. moves to the right (with velocity v), at rest, neither
   c. moves to the right, moves to the right, hits front wall
2. a. mg, down
   b. mg, up
   c. no
   d. yes
3. a. $a = \dfrac{F}{m_1 + m_2}$
   b. $m_2 a$
   c. $F - m_2 a = m_1 a$
   d. $\left(\dfrac{m_1}{m_1 + m_2}\right)F$
4. a. $a = \dfrac{F - F_k}{m_1 + m_2}$
   b. $m_2 a - F_k$
   c. $F - m_2 a - F_k = m_1 a - F_k$
   d. $\left(\dfrac{m_1}{m_1 + m_2}\right)(F - F_k)$

Chapter 5 Mixed Review

1. a. 60 J
   b. –60 J
2. a. mgh
   b. mgh
   c. $v_B = \sqrt{v_A^2 + 2gh}$
   d. no
   e. no
3. a. 2.9 J
   b. 1.8 J
   c. 1.2 J
   d. a, b: different; c: same
4. a. $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f + F_k d$
   b. $F_k = \mu mg(\cos 23°)$
   c. $v_f = \sqrt{mv_i^2 + 2g(d \sin 23° - \mu m \cos 23°)}$

Chapter 6 Mixed Review

1. a. The change due to the bat is greater than the change due to the mitt.
   b. The impulse due to the bat is greater than the impulse due to the mitt.
   c. Check student diagrams. Bat: vector showing initial momentum and a larger vector in the opposite direction showing impulse of bat, result is the sum of the vectors. Mitt: vector showing initial momentum and an equal length vector showing impulse of mitt, result is the sum, which is equal to zero.
2. a. The impulses are equal, but opposite forces, occurring during the same time interval.
   b. The total force on the bowling ball is the sum of forces on pins. The force on the pins is equal but opposite of total force on ball.
3. $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$;
   $m_1 v_{1i}/(m_1 + m_2) + m_2 v_{2i}/(m_1 + m_2) = v_f$
4. a. $M(6 \text{ m/s})$
   b. 2 m/s
   c. objects trade momentum; if masses are equal, objects trade velocities

Chapter 7 Mixed Review

1. a. 3.0, 3.0, 9.0, 27
   b. 4.3, 1.0, 4.3, 37
   c. 16, 0.28, 11, 6.0×10^2
   d. 630, 0.11,74, 8.7
   e. 5.0, 44, 0.11, 9.9
2. a. friction
   b. gravitational force
   c. tension in string
3. a. doubled
   b. quadrupled
   c. reduced to $\frac{1}{4}$
   d. quadrupled
   e. reduced to $\frac{1}{9}$
4. 190 m
5. Student diagrams should show vectors for weight and normal force from elevator; descent should show normal force less than weight; stopping should show normal force greater than weight; "weightlessness" feeling is due to acceleration.
6. 1050 s (17.5 min)

Chapter 8 Mixed Review

1. a. If the knob is farther from the hinge, torque is increased torque for a given force.
   b. twice as much
2. a. Rotational inertia is reduced.
   b. Angular momentum remains the same.
   c. Angular speed increases.
3. a. 2.0 kg
   b. 0.67 kg
4. a. 6.2 N•m, 0.016 kg•m^2, 390 rad/s^2
   b. 12 N•m, 0.062 kg•m^2, 190 rad/s^2
5. a. 8.1×10^{16} kg•m^2
   b. 5.9×10^{12} kg•m^2/s
   c. 2.1×10^8 J
   d. 3.1×10^3 m/s
   e. 2.2×10^8 J
6. a. 4.0×10^4 J
   b. 4.4×10^4 J
   c. 4.9×10^4 J
   d. 0.81

Chapter 12 Mixed Review

1. a. 0.20 s; 5.0 Hz
   b. same, same, increase, increase
2. a. 60.0 N/m
   b. 0.574 seconds; 1.74 Hz
3. 6.58 m/s^2; no
4. a. A: 0 s, 2 s, 4 s; B: 0.5 s, 1.5 s, 2.5 s, 3.5 s; C: 1, 3 s
   b. PE: 0 s at A, 1 s at C, 2 s at A, 3 s at C, 4 s at A; KE: 0.5 s, 1.5 s, 2.5 s, 3.5 s at B
   c. 0.5 s, 2.5 s at B to the right 1.5 s, 3.5 s at B to the left; 0 s, 2 s, 4 s at A to the right, 1 s, 3 s at C to the left
5. 3.00×10^2 m/s
6. 3.0 s; 6.0

Chapter 13 Mixed Review

1. a. 2.19 m; 2.27 m
   b. wavelength increases when temperature increases
2. a. arrows pointing East on ambulance, police, and truck, West on van.
   b. police and ambulance (equal), truck, small car, van
3. These objects had the same natural frequency of 330 Hz, so resonance occurred.
4. a. 1460 Hz, 2440 Hz
   b. 70.8 cm, 23.6 cm, 14.1 cm
   c. 0.177 m
   d. 974 Hz, 1460 Hz; 70.8 cm, 35.4 cm, 23.6 cm; 0.354 m
5. a. 5
   b. 435 Hz, because it will also provide a difference of 5 Hz.

Chapter 14 Mixed Review

1. 4.07×10^{16} m
2. a. 3.33×10^{-5} s
   b. 1.00×10^{-4} m
3. 3.84×10^8 m
4. 3.00×10^{11} Hz
5. Diffuse reflection: (nonshiny surfaces) table top, floor, walls, car paint, posters (answers will vary)
   Specular reflection: metallic surfaces, water, mirrors (answers will vary)
6. a. Check student drawings for accuracy.
   b. *B* is 4 m from *A* horizontally, *C* is 2 m below *B* vertically
   c. *D* is 2 m below *A* vertically, *E* coincides with *C*
   d. they will overlap the existing images or objects
7. a. 9.00 cm
   b. $p = 30.0$ cm; $q = 12.9$ cm; real; inverted; 2.58 cm tall
   $p = 24.0$ cm; $q = 14.4$ cm; real, inverted; 3.60 cm tall
   $p = 18.0$ cm; $q = 18.0$ cm; real; inverted; 6.00 cm tall
   $p = 12.0$ cm; $q = 36.0$ cm; real; inverted; 2.00 cm tall
   $p = 6.0$ cm; $q = -18$ cm; virtual; upright; 18 cm tall
8. $p = 30.0$ cm; $q = -6.92$ cm; virtual; upright; 1.38 cm tall
   $p = 24.0$ cm; $q = -6.55$ cm; virtual, upright; 1.64 cm tall
   $p = 18.0$ cm; $q = -6.00$ cm; virtual; upright; 2.00 cm tall
   $p = 12.0$ cm; $q = -5.14$ cm; virtual; upright; 2.57 cm tall
   $p = 6.0$ cm; $q = -3.6$ cm; virtual; upright; 3.6 cm tall

Chapter 15 Mixed Review

1. a. Ray 1 at 45°; Ray 2 at 14.9°
   b. Rays should intersect inside the aquarium.
   c. Because the rays are no longer parallel, they will intersect in the water.
2. a. First boundary: 70.0°, 45.0°
   Second boundary: 45.0°, 40.4°
   Third boundary: 40.3°, 36.8°
   b. Incoming rays get closer and closer to the normal. Reflected rays get farther away from the normal with the same angles.
3. a. 9.00 cm
   b. 12.9 cm, 14.4 cm, 18.0 cm, 36.0 cm, −18.0 cm
   2.58 cm, 3.6 cm, 6.00 cm, 18.0 cm, −18.0 cm
   real, real, real, real, virtual
4. 18.0 cm, with all images virtual and on the left of the lens
   −11.2, −10.3, −9.00, −7.20, −4.50
5. a. 6.00 cm in front of the lens
   b. 0.857 cm

Chapter 16 Mixed Review

1. a. 6.74×10^{-6} m
   b. 47.9°
   c. The maximum angle for light to reach the screen in this arrangement is 45°.
2. a. Longer wavelengths are diffracted with a greater angle.
   b. First order group of lines: blue, green, red; second order: the same
   c. White
3. a. A = 5.0×10^{-6} m, B = 1.1×10^{-7} m, C = 3.3×10^{-8} m
   b. visible: A; x-ray: A, B, or C; IR: none
4. a. Neither would work because they would act as different sources, so even with the same frequency, they should not be in phase.
   b. Interference is occurring.

Chapter 17 Mixed Review

1. a. A; 1.87×10^{13} electrons; B: 3.12×10^{13} electrons
   b. the forces are equal and opposite, no
2. a. Resultant = 1.49 N, left; *F*(A-C) = 1.35 N, left; *F*(B-C) = 0.140 N, left
   b. Resultant = 0.788 N, right; *F*(A-C) = 1.35 N, right; *F*(B-C) = 0.562 N, left
   c. Resultant = 0.400 N, left; *F*(A-C) = 0.599 N, right; *F*(B-C) = 0.999 N, left
3. a. 1.92×10^{16} N
   b. 2.87×10^{10} m/s^2
   c. 9.81 m/s^2; this is negligible in comparison with the acceleration *a;* alpha particles will move horizontally
4. a. Check students diagrams for accuracy.
   b. 1.53×10^{-2} N
   c. 7.65×10^3 N/C
5. 1 C = 6.25×10^{18}; 1 mC = 6.25×10^{12}

Chapter 19 Mixed Review

1. a. *I* increases because *R* decreases (shorter)
   b. no change
   c. *I* decreases because *R* increases with temperature
   d. *I* decreases
2. a. 4.8 A
   b. 8.64×10^4 J
   c. 580 W
   d. 1.0×10^7 J
3. a. 0.833 A; 1.25 A; 2.08 A
   b. 144 Ω; 96.0 Ω; 57.6 Ω
   c. 70.0 ¢; \$1.05; \$1.75
4. a. 144 V
   b. 864 W
   c. 104 seconds

Chapter 20 Mixed Review

1. a. D
   b. switch 5
   c. • switches 1 and 3 open, switches 2, 4, and 5 closed
   • switches 1 and 4 open, switches 2, 3, and 5 closed
   • switch 2 open, switches 1, 3, 4, and 5 closed; or switches 3 and 4 open, switches 1, 2, and 5 closed; or switches 2, 3, and 4 open, switches 1 and 5 closed
2. a. Check students' diagrams, which should show a bulb and a resistor in series with a battery.
   b. 15 Ω
   c. 6 Ω
3. a. Check students diagrams.
   b. 12.0 V, 12.0 V
   c. 0.25 A, 2.25 A
   d. 5.33 Ω
4. a. $R = 6.15$ Ω
   b. $R = 30.4$ Ω

Chapter 21 Mixed Review

1. a. The magnetic field from the leftmost segment is • and stronger. The magnetic field from the rightmost segment is × and weaker.
   b. At A, both horizontal segments contribute a × magnetic field of equal strength
   c. B; ×; × weaker; ×; × same
   C; ×; × same; ×; × same
   D; ×; × stronger; ×; × same
   E; ×; • stronger; ×; × same
   d. No. They reinforce each other in the same direction.
   e. inside
2. a. F = 4.3 N into the page
   b. F = 0
3. a. Diagrams should show clockwise current.
   b. Starting from the left side: F = 1.1 N into the page; F = 0; F = 1.1 N out of the page; F = 0
   c. Forces are equal and opposite, so no translational motion will occur, but it could rotate around a vertical axis.

Chapter 22 Mixed Review

1. e
2. a. 0.50 s
   b. 0.26 m^2
   c. 2.6 V
3. a. magnetic field, conductor, relative motion
   b. answers may vary, but could include the following: water wheel, windmill, electric motor, combustion engine
4. a. 6.28 rad/s
   b. 7.1×10^{-2} m^2
   c. 110 V
   d. 78 V
5. A motor converts electric energy to rotational energy; generators converts rotational energy to electric energy.
6. a. increases
   b. induces current while change occurs
   c. It decreases magnetic field which will induce a current while the change occurs.