Physics Review Notes

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The most recent version of this can be found at http://www.tomstrong.org/physics/

These notes are meant to be a summary of important points covered in the Physics class at Mt. Lebanon High School. They are not meant to be a replacement for your own notes that you take in class, nor are they a replacement for your textbook.

This is a work in progress and will be changing and expanding over time. I have attempted to verify the correctness of the information presented here, but the final responsibility there is yours. Before relying on the information in these notes please verify it against other sources.

Chapter 1 — About Science

1.1 The Basic Science — Physics

Physics is the most basic of the living and non-living sciences. All other sciences are built on a knowledge of physics. We can understand science in general much better if we understand physics first.

1.2 Mathematics — The Language of Science

Physics equations are systems of connections following all of the rules of logic. They not only tell us what is connected to what, they tell us what we can ignore. Mathematical equations are unambiguous, they don't have the multiple meanings that often confuse the discussions of ideas expressed in common language.

1.3 The Scientific Method

The scientific method is a process that is extremely effective in gaining, organizing, and applying new knowledge. The general form of the scientific method is:

1. Recognize a problem through observation.
2. Make an educated guess (a hypothesis) about the answer.
3. Predict the consequences of the hypothesis.
4. Perform one or more experiments to test the hypothesis.
5. Analyze and interpret the results of the experiment(s)
6. Share your conclusions with others in a way that they can independently verify your results.

There are many ways of stating the scientific method, this is just one of them. Some steps above are sometimes combined, at other times one step may be divided into two or more. The important part is not the number of steps listed or their grouping but is instead the general process and the emphasis on deliberate reproduceable action.

1.4 The Scientific Attitude

In science a **fact** is just a close agreement by competent observers who make a series of observations of the same phenomenon. In other words, it is what is generally believed by the scientific community to be true.

A **hypothesis** is an educated guess that is only presumed to be factual until verified or contradicted through experiment.

When hypotheses are tested repeatedly and not contradicted they may be accepted as fact and are then known as **laws** or **principles**.

If a scientist finds evidence that contradicts a law, hypothesis, or principle then (unless the contradicting evidence turns out to be wrong) that law, hypothesis, or principle must be changed to fit the new data or abandoned if it can not be changed.

In everyday speech a theory is much like a hypothesis in that it generally indicates something that has yet to be verified. In science a **theory** is instead the result of well-tested and verified hypotheses about the reasons for certain observed behaviors. Theories are refined as new information is obtained.

Scientific facts are statements that describe what happens in the world that can be revised when new evidence is found, scientific theories are interpretations of the facts that explain the reasons for what happens.

1.5 Scientific Hypotheses Must Be Testable

When a hypothesis is created it is more important that there be a means of proving it wrong than there be a means of proving it correct. If there is no test that could disprove it then a hypothesis is not scientific.

1.6 Science, Technology, and Society

Science is a method of answering theoretical questions, technology is a method for solving practical problems. Scientists pursue problems from their own interest or to advance the general body of knowledge, technologists attempt to design, create or build something for the use or enjoyment of people.

1.7 Science, Art, and Religion

Science, art, and religion all involve the search for order and meaning in the world, and while they each go back thousands of years and overlap in several ways, all three exist for different purposes. Art works to describe the human experience and communicate emotions, science describes natural order and predicts what is possible in nature, and religion involves nature's purpose.

Accuracy vs. Precision

- **Accuracy** describes how close a measured value is to the true value of the quantity being measured

  Problems with accuracy are due to error. To avoid error:

  - Take repeated measurements to be certain that they are consistent (avoid human error)
  - Take each measurement in the same way (avoid method error)
  - Be sure to use measuring equipment in good working order (avoid instrument error)

Counting Significant Figures in a Number

Rule	Example
All counted numbers have an infinite number of significant figures	10 items, 3 measurements
All mathematical constants have an infinite number of significant figures	$1/2$, π, e
All nonzero digits are significant	42 has two significant figures; 5.236 has four
Always count zeros between nonzero digits	20.08 has four significant figures; 0.00100409 has six
Never count leading zeros	042 and 0.042 both have two significant figures
Only count trailing zeros if the number contains a decimal point	4200 and 420000 both have two significant figures; 420. has three; 420.00 has five
For numbers in scientific notation apply the above rules to the mantissa (ignore the exponent)	4.2010×10^{28} has five significant figures

Counting Significant Figures in a Calculation

Rule	Example
When adding or subtracting numbers, find the number which is known to the fewest decimal places, then round the result to that decimal place.	$21.398 + 405 - 2.9 = 423$ (3 significant figures, rounded to the ones position)
When multiplying or dividing numbers, find the number with the fewest significant figures, then round the result to that many significant figures.	$0.049623 \times 32.0/478.8 = 0.00332$ (3 significant figures)
When raising a number to some power count the number's significant figures, then round the result to that many significant figures.	$5.8^2 = 34$ (2 significant figures)
Mathematical constants do not influence the precision of any computation.	$2 \times \pi \times 4.00 = 25.1$ (3 significant figures)
In order to avoid introducing errors during multi-step calculations, keep extra significant figures for intermediate results then round properly when you reach the final result.	

- **Precision** refers to the degree of exactness with which a measurement is made and stated.
  - 1.325 m is more precise than 1.3 m
  - lack of precision is usually a result of the limitations of the measuring instrument, not human error or lack of calibration
  - You can estimate where divisions would fall between the marked divisions to increase the precision of the measurement

Chapter 2 — Linear Motion

2.1 Motion is Relative

To measure the motion of an object it muse be measured **relative** to something else, in other words some reference needs to be there to measure the motion against. You can measure the motion of a car moving down the highway, but if you measure it from another moving car you will get very different measurements than if you are a person standing beside the road.

2.2 Speed

A moving object travels a certain distance in a given time. **Speed** is a measure of how fast something is moving or how fast distance is covered. It is measured in terms of length covered per unit time so you will encounter units like meters per second or miles per hour when working with speed.

Instantaneous Speed

The speed that an object is moving at some instant is the **instantaneous speed** of that object. If you are in a car you can find the instantaneous speed by looking at the speedometer. The instantaneous speed can change at any time and my be changing continuously.

Average Speed

The average speed is a measure of the total distance covered in some amount of time. In the car from the example above the average speed is the total distance for the trip divided by the total time for the trip, the car could have been traveling faster or slower than that speed during parts of the trip, the average is only concerned with the total distance and the total time. Mathematically, you can find the average speed as

$$average\ speed = \frac{total\ distance\ covered}{time\ interval}$$

2.3 Velocity

In everyday language **velocity** is understood to mean the same as speed, but in physics there is an important distinction. Speed and velocity both describe the rate at which something is moving, but in addition to the rate velocity also includes the direction of movement. You could describe the speed of a car as 60 km per hour, but the car's velocity could be 60 km per hour to the north, or to the south, or in any other direction so long as it is specified. You can change an object's velocity without changing its speed by causing it to move in another direction but changing an object's speed will always also change the velocity.

2.4 Acceleration

The rate at which an object's velocity is changing is called its **acceleration**. It is a measure of how the velocity changes with respect to time, so

$$acceleration = \frac{change\ in\ velocity}{time\ interval}$$

Be sure to realize that it is the change in velocity, not the velocity itself that involves acceleration. A person on a bicycle riding at a constant velocity of 30 km per hour has zero acceleration regardless of the time that they ride as long as the velocity remains constant.

Acceleration in a negative direction will cause an object to slow down, this is often referred to as deceleration or negative acceleration.

2.5 Free Fall: How Fast

When something is dropped from a height it will fall toward the earth because of gravity. If there is no air resistance (something that we will usually assume) then the object will fall with a constant acceleration and be in a state known as **free fall**.

If you were to start a stopwatch at the instant the object was dropped the stopwatch would measure the **elapsed time** for the fall.

All objects in free fall will fall with the same acceleration, that is the acceleration caused by gravity and it is given the symbol g. A precise value would be $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$ but for most purposes we will make the math a bit easier and use $g = 10\ \frac{\mathrm{m}}{\mathrm{s}^2}$. (Use $g = 10\ \frac{\mathrm{m}}{\mathrm{s}^2}$ unless you are told otherwise.)

For an object starting at rest the relationship between the instantaneous velocity of the object (v) and the elapsed time (t) is

$$v = gt$$

This equation will only work for an object dropped from rest, if the object is thrown upward or downward then the problem has to be broken up into smaller pieces, taking advantage of the relationship between the speed of the object at the same elevation as shown on page 18 of your textbook.

2.6 Free Fall: How Far

If an object is in free fall and dropped from rest then the relationship between the distance the object has fallen (d), the elapsed time (t), and the gravitational acceleration constant (g) is

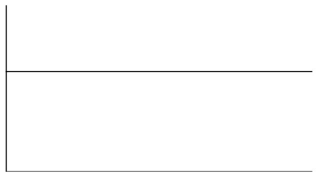
$$d = \frac{1}{2}gt^2$$

The mathematical models for free fall will also work for anything else moving in a straight line with constant acceleration, all you need to do is replace the gravitational acceleration constant (g) with the acceleration of the object you are studying (a)

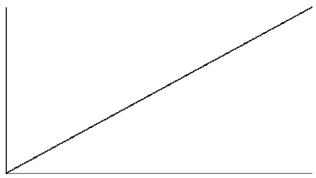
$$v = at \qquad d = \frac{1}{2}at^2$$

2.7 Graphs of Motion

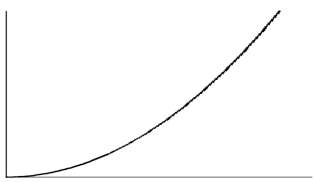
For an object starting at rest and moving with a constant acceleration a typical set of graphs of acceleration, velocity, and distance traveled would look like this when graphed with respect to time



Acceleration — a is a constant



Velocity — $v = at$



Distance — $d = \frac{1}{2}at^2$

At every point the value of the velocity graph is the slope of the distance graph, and the value of the acceleration graph is the slope of the velocity graph. Similarly, at every point the velocity graph is equal to the area under the acceleration graph to that point and the displacement graph is equal to the area under the velocity graph to that point.

2.8 Air Resistance & Falling Objects

When actual objects are dropped air resistance will cause different objects to fall in slightly different ways. For example, a brick will fall about the same way whether there is air resistance or not since it is heavy and compact but a piece of paper will tend to float downward. Wadding up the paper into a ball will cause it to fall faster by decreasing the area that has to move through the air but it will still fall slower than a brick dropped at the same time.

2.9 How Fast, How Far, How Quickly How Fast Changes

Velocity is the rate of change of how far something has traveled, acceleration is the rate of change of the velocity or the rate of change of the rate of change of the distance. This may sound confusing at first, if so back up and look at it a piece at a time as it was initially explained in the earlier parts of the chapter.

Chapter 3 — Projectile Motion

3.1 Vector & Scalar Quantities

A **vector** is something that requires both **magnitude** (size) and direction for a complete description. This could be how far a rock has fallen (20 m downward), how quickly a car is accelerating (10 meters per second per second forward), how hard something is being pushed (with 35 newtons of force to the left), or any number of other quantities.

If something can be completely described with just magnitude alone then it is known as a **scalar** quantity. Examples of scalars could be an elapsed time (22 seconds), a mass (15 kilograms) or a volume (0.30 cubic meters). Trying to add a direction to a scalar quantity (15 kilograms to the right) would not have any useful meaning in physics.

3.2 Velocity Vectors

When vectors are drawn they are customarily drawn so that their length is proportional to the magnitude of the quantity they represent. If a group of vectors are being added their sum (the **resultant**) can be found by drawing each of the vectors to scale and in the proper direction so that one vector starts where the previous one ends. If everything is drawn to scale then the resultant will just be another vector drawn from the beginning of the first vector to the end of the last one. (See pages 30 and 31 of your textbook for examples of how this works)

3.3 Components of Vectors

Just as any two (or more) vectors representing the same quantity (all velocity, all force, etc. — you can't add a force to a velocity) can be added to find a single resultant vector you can also take any single vector and break it into two pieces that are at right angles to each other. This is called **resolution** of the vector into **components**. This can be done by drawing the vector to scale and at the proper angle, finding a rectangle that will just fit around it, and then measuring the sides of the rectangle (as shown in your book on page 31) or you can do it mathematically using sine and cosine if you prefer.

3.4 Projectile Motion

Any object that is shot, thrown, dropped, or otherwise winds up moving through the air (or even above the air in some cases) is known as a **projectile**. The horizontal and vertical motion of a projectile are independent of each other, the horizontal motion is just motion with a constant velocity, vertically it is just free fall. Pages 33 and 34 of your textbook have several examples of this type of motion.

3.5 Upwardly Launched Projectiles

Because of gravity anything launched upward at an angle will follow a curved path and (usually, unless it is moving extremely fast) return to the earth. If there was no gravity then if you threw a softball it would travel forever in a straight line, because of gravity the softball will fall away from that straight line just as much as if it was dropped. (After 1 second it will be 5 m below the line, after 2 seconds it will be 20 m below it, after 3 seconds 45 m, etc.)

The components of the velocity will change in very different ways as the projectile moves. The horizontal component will not change at all, the vertical component will be the same as for another object thrown straight up. You can take advantage of this to find how long something will be in the air from one of the components, then, using the time that you just found, find the other missing pieces of the problem.

An object thrown at 45° will travel farther than at any other angle, one thrown straight up will reach a higher maximum height.

For an object launched with an initial velocity v_i at an angle of θ above the ground the distance it will travel is

$$d = \frac{v_i^2 \sin(2\theta)}{g}$$

where g is just the familiar gravitational acceleration constant.

3.6 Fast-Moving Projectiles — Satellites

If something is thrown extremely fast (faster than about 8 km per second) then it will never fall to the earth, instead the earth will curve away faster than the object will fall and it will go into orbit as a **satellite**.

Chapter 4 — Newton's First Law of Motion — Inertia

4.1 Aristotle on Motion

The Greek scientist Aristotle divided motion into natural motion (objects moving straight up or straight down, heading toward their eventual resting place) and violent motion (motion imposed by an external cause moving an object away from its resting place). According to Aristotle it is the nature of any object to come to rest and the object will do this on its own, it does not need any external influence for this to happen.

4.2 Copernicus and the Moving Earth

Copernicus reasoned that the simplest way to explain astronomical observations was to assume that the the Earth and other planets moved around the sun instead of the common belief that the Earth was at the center off the universe. This was contrary to Aristotle's teachings which were widely accepted at the time and caused Copernicus to delay the publication of his findings until almost the end of his life.

4.3 Galileo on Motion

On of Galileo's contributions to physics was the idea that a **force** is not necessary to keep an object moving.

A force is any push or pull on an object. **Friction** is the force that occurs when two objects rub against each other and the small surface irregularities create a force that opposes the moving object(s). Galileo argued that only when friction is present is a force necessary to keep an object moving. The **inertia** of an object is a measure of its tendency to keep moving as it is currently moving.

Galileo studied how objects moved rather than why. He showed that experiments instead of logic were the best test of ideas.

4.4 Newton's Law of Inertia

Newton's first law, often called the **inertia law** states *every object continues in a state of rest, or of motion in a straight line at constant speed, unless it is compelled to change that state by forces exerted upon it.*

Another way of saying this is that objects will continue to do whatever they are currently doing unless something causes them to change. No force is required to maintain motion, a force is needed only to change it.

4.5 Mass — A Measure of Inertia

Mass is the amount of matter in an object or more specifically a measure of the inertia of an object that will resist changes in motion.

The **weight** (F_g) of an object is the gravitational force upon it, it can be found with the equation

$$weight = mass \times gravitational\ acceleration$$

or

$$F_g = mg$$

4.6 Net Force

The **net force** acting on an object is the sum of all of the forces acting on it. They may cancel each other out either totally or partially or they may combine to produce a force greater than any of the individual forces on the object.

4.7 Equilibrium — When Net Force Equals Zero

When the net force on an object is zero the object is in a state of **equilibrium**. This could be because all of the forces are canceling each other out or it could be because there is actually no force on the object, either way the object will not accelerate.

4.8 Vector Addition of Forces

Forces are added as vectors just as displacement, velocity, and acceleration are. The same methods of vector addition will work that you have used previously. Your textbook has several examples of vector addition of forces on pages 52–54.

4.9 The Moving Earth Again

All measurements of motion are made relative to the observer. If you are in a moving car you can look at objects in the car that appear stationary to you but in fact the are moving with you. The same thing happens with respect to the Earth. Since the Earth moves everyone on it will move with it and we will not notice that motion, everything moves on the Earth as if the Earth was not moving.

Chapter 5 — Newton's Second Law of Motion — Force and Acceleration

5.1 Force Causes Acceleration

If the net force acting on an object is no zero the object will be accelerated, the acceleration of the object will be directly proportional to the net force on the object — if the net force is doubled the acceleration will also be doubled.

5.2 Mass Resists Acceleration

The larger the mass of an object the smaller the acceleration that will be produced by a constant net force. The exact relationship is that the acceleration is inversely proportional to the mass, another way of saying that is that the acceleration is proportional to one divided by the mass of the object.

5.3 Newton's Second Law

Newton's second law states that *the acceleration produced by a net force on an object is directly proportional to the magnitude of the net force, is in the same direction as the net force, and is inversely proportional to the mass of the object.*

Mathematically, if F is the net force, Newton's second law can be expressed as

$$F = ma \qquad \text{or} \qquad a = \frac{F}{m}$$

5.4 Friction

Friction is a force that acts between any two objects in contact with each other to oppose their relative motion. The force of friction depends on the surfaces in contact (rougher surfaces produce more friction) as well as the force pressing the surfaces together (the more force pressing the surfaces together the more friction there will be). When frictional forces are present they must be accounted for along with other applied forces to calculate the net force on objects.

5.5 Applying Force — Pressure

When two objects exert force on each other the **pressure** on the surface in contact can be found as

$$pressure = \frac{force}{area\ of\ contact}$$

The pressure can be increased by applying more force over the same area or by applying the same force over a smaller area.

5.6 Free Fall Explained

The weight of an object is the force that causes an object to fall, and the weight is the product of the mass and the gravitational acceleration constant. Since $F = ma$ the acceleration is equal to the force divided by the mass, this yields:

$$a = \frac{F}{m} = \frac{mg}{m} = \frac{\cancel{m}g}{\cancel{m}} = g$$

In the absence of air resistance or other frictional forces the acceleration of a falling object is equal to g, the gravitational acceleration constant.

5.7 Falling and Air Resistance

At low speeds air resistance (R) is very small and can usually be ignored for most objects, as the speed increases so does the air resistance. The heavier and smaller the object (such as a steel ball) the less air resistance will affect it, larger and lighter objects (such as a feather or a piece of paper) will be affected more because of the combination of a larger area and a smaller mass.

Chapter 6 — Newton's Third Law of Motion — Action and Reaction

6.1 Forces and Interactions

A force is any push or pull on an object, but it also involves a second object to produce the force. This mutual action, or **interaction** actually causes there to be a pair of forces — if a hammer applies a force to a nail the nail will also apply a force back on the hammer.

6.2 Newton's Third Law

Newton's third law states *whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first object.*

One force is called the **action force**, the other force is called the **reaction force.** Neither force can exist without the other and they are equal in strength and opposite in direction. Newton's third law is often expressed as "for every action there is an equal and opposite reaction".

6.3 Identifying Action and Reaction

When analyzing forces the action force may be apparent but the reaction force is often harder to see until you know what to look for. Whatever the action force is, the reaction force always involves the same two objects. If the action force is the Earth pulling down on you with your weight the reaction force would be you pulling up on the Earth with the same force. The two objects do not have to be in contact to interact.

6.4 Action and Reaction on Different Masses

Action and reaction forces always act on a pair of objects. When a rifle is fired the action force is the rifle acting on the bullet, the reaction force is the bullet pushing back on the rifle. Since the bullet has a small mass and the rifle has a large mass the bullet will have a larger acceleration.

6.5 Do Action and Reaction Forces Cancel?

Since action and reaction forces each act upon different objects they can never cancel each other. In order to cancel each other out forces must all act upon the same object.

6.6 The Horse-Cart Problem

See the explanation and illustrations on pages 80–82 of your textbook for an excellent explanation of how action-reaction pairs work and how to handle multiple force pairs in one problem.

6.7 Action Equals Reaction

In order to apply a force to an object that object must be able to apply an equal and opposite force to you. The example that they use is trying to apply a 200 N punch to a piece of paper — when you apply even a small force to the paper it will easily accelerate out of your way from the applied force. Since it can't stay in one place to apply a reaction force you cannot apply the larger action force.

Chapter 7 — Momentum

7.1 Momentum

Momentum is a vector quantity described by the product of an object's mass times its velocity:

$$p = mv$$

Momentum is also known as inertia in motion. An object must be moving to have momentum, objects at rest have none.

7.2 Impulse Changes Momentum

A force acting on an object for some amount of time produces an **impusle** which will change the object's momentum. The impulse is defined as the product of the force and the time the force acts, so

$$impulse = Ft$$

Since impulse is also the change in momentum this then becomes

$$\Delta p = Ft$$

The same impulse can be delivered by a small force acting over a long time or a large force acting for a short time. If an object will be brought to a stop increasing the time during which the force acts will cause the force to be reduced, this is the principle behind padding, air bags, car bumpers, and many other safety devices.

7.3 Bouncing

When an object is brought to a stop the change in momentum is equal to the momentum that the object originally had. If the object bounces then the change in momentum will be twice the original momentum of the object, so an object that bounces will deliver twice the impulse of one that comes to a stop.

7.4 Conservation of Momentum

When two objects interact the momentum lost by one object will be gained by the other. No momentum will be created or destroyed, it will only be moved around between the objects. When momentum (or any other quantity, such as mass or energy) acts like this we say that it is **conserved**. This leads to one of the principla laws of mechanics, the **law of conservation of momentum** which states *In the absence of an external force, the momentum of a system remains unchanged.* The external force being referred to in this case is one coming from an object not in the system.

7.5 Collisions

When two objects collide we will consider two cases, one where the two objects bounce off of each other (known as an **elastic collision**) and another where they stick together and travel as a single combined object (known as an **inelastic collision**). In either case since momentum is conserved the momenta of the two objects before and after the collision will follow the relationship

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

7.6 Momentum Vectors

Since momentum is a vector quantity it can act in any number of directions, not just in a straight line. The diagrams in your textbook on page 98 help to illustrate what will happen in a collision when the two objects colliding do not both move in the same line.

Chapter 8 — Energy

8.1 Work

When a force acts on an object over some distance (moving the object parallel to the direction of the force) the force is said to do **work** (W) on the object, the amount of work done is the product of the force times the distance over which the force was applied:

$$W = Fd$$

Work is measured in newtons times meters which are given the name **joules** (J).

8.2 Power

Power (P) is defined to be the rate at which work is done, or the work done divided by the time interval during which the work was done:

$$P = \frac{W}{t}$$

Power is measured in joules divided by seconds which are given the name **watts** (W). One watt of power is what is required to convert one joule of energy every second.

8.3 Mechanical Energy

Energy is the accumulation of work done in an object. If work is done in making an object move faster it has the ability to do work on something else when it slows down (**kinetic energy**), if the work is done in raising it to some height then it has the ability to do work as it is lowered back to its original position (**potential energy**). Just like work energy is measured in joules.

8.4 Potential Energy

Potential energy (PE) is the energy stored in an object by lifting it to a higher position, the force required to lift the object it its weight (mg) and the distance over which that force acts is the height (h), so the product of them is the potential energy:

$$PE = mgh$$

Potential energy is a relative measurement, you choose the height that will be your zero level when you set up your frame of reference.

8.5 Kinetic Energy

Kinetic energy (KE) is the energy an object has because it is moving and it is equal to the work done in accelerating the object to the speed at which it is traveling. The more mass or speed the object has the larger the kinetic energy:

$$KE = \frac{1}{2}mv^2$$

8.6 Conservation of Energy

Energy can be transferred from one object to another by one of them doing work on the other, it can also be transferred from one kind of energy to another within an object. This leads to the **law of conservation of energy** which states that *energy cannot be created or destroyed. It can be transformed from one form into another, but the total amount of energy never changes.* If anything exerts a force on an object in a system then the source of that force must also be included in the system for the law of conservation of energy to hold, otherwise energy could be moved into or out of the system.

For a single object moving without friction or other interactions the law of conservation of energy states that the sum of the kinetic and potential energies for the object will always be the same, so

$$PE_i + KE_i = PE_f + KE_f$$

or, by expanding those terms

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

8.7 Machines

A **machine** is a device that is used to multiply forces or change the direction of forces. All machines are subject to the law of conservation of energy, no machine can produce more energy than is fed into it. In the ideal case no work will be lost to friction so

$$W_{out} = W_{in}$$

And since $W = Fd$ this can become

$$F_{out}d_{out} = F_{in}d_{in}$$

The amount that the machine multiplies the input force to get the output force is called the **mechanical advantage** (MA) of the machine:

$$MA = \frac{F_{out}}{F_{in}}$$

In the absence of friction the **theoretical mechanical advantage** (TMA) can also be expressed as the ratio of the input distance to the output distance

$$TMA = \frac{d_{in}}{d_{out}}$$

Some common machines are shown in your textbook on pages 112-115.

8.8 Efficiency

The **efficiency** of a machine is the ratio of work output to work input, or

$$eff = \frac{W_{out}}{W_{in}}$$

It can also be expressed as the ratio between the actual mechanical advantage (MA) and the theoretical mechanical advantage (TMA):

$$eff = \frac{MA}{TMA}$$

8.9 Energy for Life

Living cells can also be regarded as machines, they take energy from either fuel or the sun and convert it to other forms.

Chapter 9 — Circular Motion

9.1 Rotation and Revolution

If an object rotates around an **axis** that passes through the object (an internal axis) then the motion is described as **rotation**, if the axis does not pass through the object then the motion is described as **revolution**. As an example, every day the Earth rotates around its axis, once a year it revolves around the sun.

9.2 Rotational Speed

The speed of a rotating object can be measured different ways, first the **rotational speed** (also known as **angular speed**, ω) measures how many rotations occur in a unit of time, for a rigid body (something that will not change shape as you are studying it) every point on the object will have the same angular speed. The other type of speed is the **tangential speed** (v), that measures how fast a single point on the object is moving when measured along a straight line that is tangent to the motion of that point. The farther an object is from the axis the larger the tangential speed will be, a point on the axis will have a tangential speed of zero. The two speeds are related through the equation

$$\textit{tangential speed} = \textit{radial distance} \times \textit{angular speed}$$

or

$$v = r\omega$$

9.3 Centripetal Force

If an object is moving with no net force acting on it then Newton's first law says it will move in a straight line. To keep an object moving in a circular path requires a constant force toward the center known as a **centripetal force** (a center-seeking force). For an object of mass m moving in a circle of radius r with tangential speed v and angular speed ω the centripetal force, F_c, is found by

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

The corresponding centripetal acceleration (a_c) is

$$a_c = \frac{v^2}{r} = r\omega^2$$

9.4 Centripetal and Centrifugal Forces

It it important to realize that the force causing circular motion is a centripetal force, not at force pulling away from the axis of rotation (a **centrifugal force** or center-fleeing force). There is no centrifugal force causing circular motion.

9.5 Centrifugal Force in a Rotating Reference

If you are in some object that is moving in a circular path you might feel like you are experiencing a centrifugal force but in actuality there is no such force, what you are experiencing is the reaction force to the object pushing you toward the center of the circle. No outward force exists on an object as a result of the object's motion in a circle.

9.6 Simulated Gravity

If a space station were constructed in the shape of a large wheel (as shown in the diagrams on page 131 of your textbook) and the wheel were rotated about the axis then anyone on the inside of the wheel would feel that they were being pulled downward as if by gravity. This is caused by the centripetal force that keeps them moving in a circle, their reaction force will seem to pull them "down" toward the outside rim of the station.

The magnitude of the force acceleration or force experienced for a given rotational speed is proportional to the radial distance to the "floor" so a larger space station would have a larger simulated gravity than a smaller one. A smaller space station would also give a greater difference in apparent gravitational force from a person's head to their feet than a larger one would.

Chapter 10 — Center of Gravity

10.1 Center of Gravity

When studying physical objects it eventually becomes necessary to consider their size and shape. Up until now we have been looking at objects as if they could be represented by a single point. If an object has definite size and shape then it should be possible to find a single point to represent the object, that point is the **center of gravity** of the object.

If an object is thrown through the air or slid across a frictionless table while it spins then it is the center of gravity of the object that will trace out a smooth curve. All of the other parts of the object will rotate around the center of gravity when it spins.

10.2 Center of Mass

Center of gravity is often also called **center of mass**, in most cases there is no difference although if an object is large enough for the gravitational attraction to vary from one side to the other (such as a very tall building) then there may be a very small difference between the two points. In this class we will often use the two terms interchangeably.

10.3 Locating the Center of Gravity

The center of gravity of a uniform object is at the geometric center of the object, for a more complicated object it can be found by suspending the object from different points, tracing a line straight down from the point of suspension, repeating this, and then seeing where the lines cross — this point of intersection is the center of gravity of the object.

The center of gravity of an object may be located where no actual material of the object exists, for example the center of gravity of a donut is in the middle of the hole.

10.4 Toppling

Every object that rests on a surface will contact that surface in one or more points, if you were to imagine a rubber band stretched around those points the area enclosed by that rubber band is the object's supporting base. As long as the center of gravity is above (or hanging below) that supporting base the object will remain in place. If the center of gravity moves to no longer be over (or under) the supporting base then the object will fall over.

10.5 Stability

An object with no net force acting on it can be either easy or difficult to tip over, depending on the size of the supporting base of the object and how close the center of gravity is to the edge of the supporting base.

If an object is placed so that its center of gravity is directly over the edge of its supporting base then any small displacement will start to lower the center of gravity and the object will tip over, this condition is known as **unstable equilibrium**.

If the object is placed so that its center of gravity is not on the edge of its supporting base then a small displacement will start to raise the center of gravity and the object will return to the initial position when released, this is known ad **stable equilibrium**.

A third condition also exists where the distance from the center of gravity to the outside of an object is the same for any (or most) orientations of the object, in that case a small displacement will neither raise nor lower the center of gravity and the object will remain in the new, displaced, position, this is known as **neutral equilibrium**.

An object or system will tend to behave in a way that will lower its center of gravity — tall objects will fall over, balls will roll down hill, water will run downward through a pipe, and heavier objects will settle to the bottom while lighter ones rise to the top in a mixture.

10.6 Center of Gravity of People

A person's center of gravity is about halfway between their front and back and a few centimeters below their navel if they are standing up straight, if they change their body position then the center of gravity will change as well. As you move around you will generally shift your body to keep your center of mass somewhere over your supporting base.

Chapter 11 — Rotational Mechanics

11.1 Torque

A force exerted some distance from the axis of rotation of an object is known as a **torque**. If a force is located on a line that passes through the axis of rotation (or the center of mass of the object if there is no fixed axis) it will cause the object to move in a straight line instead, no force passing through the axis of rotation can cause any torque.

The torque produced by a force is equal to **lever arm** (the distance from the axis of rotation to the point where the force is acting) times the component of the force that is perpendicular to the lever arm, mathematically that would be

$$torque = lever\ arm \times perpendicular\ force$$

The same torque can be produced by a large force with a short lever arm or a small force with a long lever arm.

11.2 Balanced Torques

If the sum of the torques in a clockwise direction is equal to the sum of the torques in the counterclockwise direction (a net torque of zero) then the object the torques are acting on is said to be in **rotational equilibrium**. Two people on a non-moving seesaw or a balanced scale would be examples of objects in rotational equilibrium.

See page 153 of your textbook for an example of how to use balanced torques to find unknown forces or distances.

11.3 Torque and Center of Gravity

Any force passing through the center of gravity of an object will tend to move the object instead of rotating it, if the force instead passes some distance from the center of gravity then the object will experience a torque that will tend to cause the object to rotate as well as move.

11.4 Rotational Inertia

An object's resistance to rotation is known as its **rotational inertia**, also known as **moment of inertia**. Just as an object with larger mass is more difficult to start or stop moving than one with a small mass, an object with a large moment of inertia is more difficult to start or stop rotating than one with a small moment of inertia.

The farther the mass of an object is from its axis of rotation the larger the object's moment of inertia, an object with the same mass closer to the axis of rotation will have a smaller moment of inertia. For an object of mass m with all of its mass a distance r from the axis of rotation the moment of inertia I will be

$$I = mr^2$$

Moments of inertia of more complicated objects are listed in your textbook on page 157.

Since objects with large moments of inertia are more resistant to changing their rotational speed than those with smaller moments of inertia if two or more objects are rolled down an inclined plane the first one to reach the bottom will be the one with the smallest moment of inertia, a solid cylinder will beat a ring and a sphere will beat both the cylinder and the ring.

11.5 Rotational Inertia and Gymnastics

A human body can be considered to have three principal axes as pictured on page 159 of your textbook. They are the longitudinal, transverse, and median axes. A body's moment of inertia is least around the longitudinal axis and about equal around the other two.

11.6 Angular Momentum

Just as an object moving in a straight line has momentum equal to the product of its mass times its velocity an object that is rotating has **angular momentum** equal to the product of its moment of inertia times its angular speed. Unlike linear momentum which is always the same for any mass and velocity the angular momentum of an object will vary depending on the arrangement of the object's mass in addition to the mass and angular velocity.

11.7 Conservation of Angular Momentum

Angular momentum is conserved, a rotating object will always have the same amount unless the angular momentum is transferred to some other object. This, in combination with the dependence on the arrangement of mass will allow an object to change its rotational speed if the arrangement of mass changes, a notable example of this is standing on a rotating platform while holding weights, if you start with your arms outstretched you will gain speed when you pull your arms in despite your angular momentum remaining constant.

Chapter 12 — Universal Gravitation

12.1 The Falling Apple

Newton observed that falling objects (such as the infamous apple) were just one example of gravitational attraction, there is a gravitational attraction between every pair of objects no matter their size or distance.

12.2 The Falling Moon

In the absence of some external force the moon should continue to move in a straight line. Since it does not move in a straight line but instead orbits the earth there must be some force acting on it. This turns out to be the gravitational attraction between the earth and the moon. Every second the moon falls toward the earth a very small amount relative to where it would be if it kept traveling in a straight line, that distance that the moon falls is about 1.4 mm every second.

12.3 The Falling Earth

Just as the moon falls around the earth, the earth and all of the other planets fall around the sun. If they ever stopped moving they would fall straight in toward the sun.

12.4 Newton's Law of Universal Gravitation

Newton discovered that gravitational attraction is a universal force. Every bit of matter in the universe attracts every other bit of matter, no matter how small they are or how far apart they may be. The exact force of gravitational attraction, F_g, between any two objects of masses m_1 and m_2 separated by a distance r is

$$F_g = G\frac{m_1 m_2}{r^2}$$

where G, the constant of universal gravitation, is found experimentally to be

$$G = 6.673 \times 10^{-11}\ \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$$

12.5 Gravity and Distance: The inverse Square Law

Because the denominator of the equation above includes the square of the distance gravity is said to follow the inverse square law. As the distance increases between two objects their force of attraction will decrease by the square of the amount the distance increased, for example doubling the distance will produce 1/4 of the force, tripling it will produce 1/9 of the force, etc. There is a figure on page 176 in your textbook that helps to explain this phenomenon.

12.6 Universal Gravitation

As discussed above, every particle of matter in the universe attracts every other particle. This has led to the discovery of the planet Neptune because of the observed effects of its gravitational attraction on the planet Uranus, and the same thing happened to lead to the discovery of the former planet Pluto.

Chapter 13 — Gravitational Interactions

13.1 Gravitational Fields

Gravity is a field force (a force that will act between two objects not in contact) and the strength of the gravitational field at any point is the acceleration that would be caused on an object at that point. At the surface of the earth the gravitational field strength is the familiar gravitational acceleration constant, $g = 9.8\ \frac{\text{m}}{\text{s}^2}$.

To find the gravitational field strength g at any point a distance r from the center of mass of a planet of mass m we use a modification of the universal gravitation equation

$$g = G\frac{m}{r^2}$$

where $G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

13.2 Gravitational Field Inside a Planet

The equation above for the gravitational field strength assumes that all of the mass of the planet is on one side of you, if you burrow down into the planet then some of the planet's mass will be on the other side of you, that will partially cancel the force from the rest of the mass and reduce the effect. At the center of the planet the gravitational attraction from one side would be exactly balanced by the attraction from the other side and you would feel no net gravitational force.

13.3 Weight and Weightlessness

If you are in an enclosure that is being accelerated you will feel a change in your apparent weight, think about being in an elevator that is changing speed, if it's starting to move upward you will feel heavier than normal, if it's starting to move downward you will feel lighter than normal. If somehow the supporting cables were to break then you and the elevator car would both fall at the same rate and you would feel no weight relative to the elevator.

13.4 Ocean Tides

Since the gravitational pull depends on the distance between the objects there is a difference in the gravitational pull from the moon to the near and far sides of the earth. For a solid object this would not be noticeable but for a liquid like the water in the oceans it means that the water is pulled more on the side closer to the moon and less on the side away from the moon. This gives a tidal bulge on the side toward the moon (pulled there by the moon) and another one on the side away from the moon (where there is a smaller pull from the moon and the oceans move away). There are also similar tidal bulges produced by the sun but because the sun is much farther away they are less than half the size of those produced by the moon.

13.5 Tides and the Earth and Atmosphere

The earth itself is not a completely rigid object, it's made up of a slightly flexible crust over a less solid inside. Because of this the earth's surface itself is affected by the tidal forces. It's not a noticeable effect since everything on the surface moves with the surface but it is measurable and has been found to be as much as 25 cm.

13.6 Black Holes

Stars are large accumulations of mostly gas that are constantly being affected by two forces — the gravitational force pulling the gas in toward the center and the nuclear fusion in the core that pushes the gas back out. If the nuclear fusion weakens over time the gravitational force will pull the gas inward, and if there is enough gas there to pull inward then eventually the star will collapse inward onto itself and form what is known as a black hole. (Smaller stars will form what is known as a back dwarf, effectively a cinder that is all that's left of a burned-out star.)

The amount of mass present in the black hole is exactly the same as the mass in the star that collapsed so anything that was orbiting the star will orbit the black hole instead. The difference comes with the size of the black hole — since it is much smaller than the star it is now possible to get much closer to the center than was previously possible. Getting closer will greatly increase the gravitational force in that area and eventually the pull of gravity is so large that even light can't get away from the center.

Chapter 14 — Satellite Motion

14.1 Earth Satellites

A satellite is any object that falls around the earth instead of falling down into it. Since a projectile would drop about 5 m every second and the earth's curvature is such that you have to travel about 8000 m for the surface to curve away 5 m, if you are traveling at 8000 m per second near the surface of the earth then you will never hit the earth — you will always fall around it instead. The result of this would be that instead of hitting the earth the object would instead be in a very low orbit.

As a practical matter if you are moving close to the surface of the earth then the friction of the atmosphere will quickly slow you down so most satellites stay about 150 km above the earth to avoid the atmosphere.

14.2 Circular Orbits

If an object is in a circular orbit then it is always moving at the same speed and always at the same distance from the earth. There is no component of the gravitational force in the direction of the satellite's motion so there can be no acceleration on the satellite.

The **period** of the satellite is the time that it takes for one complete orbit, if the satellite is orbiting close to the earth (within a few hundred kilometers of the surface) then the period is about 90 minutes.

A special kind of circular orbit is what is known as a **geosynchronous** or **geostationary** orbit, that is an orbit over the equator with a period of 24 hours and will cause the satellite to always be above the same point on the earth. This is used for communications satellites and others that always need to be in the same position relative to the earth so that they can be found by transmitters and receivers on the ground.

14.3 Elliptical Orbits

If a satellite close to the earth has a speed higher than 8 km per second then it will overshoot a circular orbit and move farther from the earth, trading off kinetic energy for potential energy as it slows down while increasing altitude. The path traced out will be an ellipse with the earth at one focus. (A circle is a special case of an ellipse with only one focus.)

The speed of the satellite will be smallest at the highest point (called the **apogee**) and largest at the lowest point (called the **perigee**).

14.4 Energy Conservation and Satellite Motion

The total energy (kinetic plus potential) of any satellite is constant. For an elliptical orbit one kind of energy is constantly being traded for the other, for a circular orbit both kinetic and potential energy remain constant, but in either case their sum is always the same.

14.5 Escape Speed

The **escape speed** of an object is the speed that something must travel to completely leave the earth (or another planet or star). For an object on the surface of the earth that speed is 11.2 km per second. Anything launched slower than that speed will eventually return to earth, anything faster will not. If the object starts from farther away then the escape speed will be reduced, similarly if the planet has a smaller mass then the escape will also be reduced. There is a chart in your textbook on page 208 with the escape speeds from several bodies in the solar system.

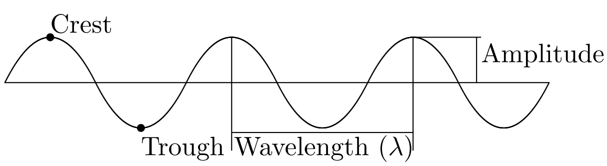
Chapter 25 — Vibrations and Waves

25.1 Vibration of a Pendulum

If you place a heavy object on the end of a light string and allow it to swing back and forth then you have a simple pendulum. Galileo discovered that for small oscillations the time it takes for a pendulum to make one complete swing back and forth depends only on the length of the pendulum, not on the mass of the pendulum bob. This amount of time to make one complete vibration is known as the **period** (T) of the pendulum. A long pendulum will have a long period, a shorter pendulum will have a shorter period.

25.2 Wave Description

The back and forth motion of a pendulum is called **simple harmonic motion.** If the position of an object in simple harmonic motion is graphed with respect to time the result will be a **sine curve.** A since curve has certain parts to it that correspond to the parts of wave motion, these are the **crests** and **troughs** (the highest and lowest points on the wave), the **amplitude** (A, the distance from the center to the extreme points in the oscillation), and the **wavelength** (λ, the distance between any two adjacent corresponding parts of waves, such as one crest to the next or one trough to the next). These are shown in this diagram:



The number of vibrations that happen per second is called the **frequency** (f) of the oscillation, this is measured in Hertz (Hz) or inverse seconds (s^{-1}). The relationship between period and frequency is an inverse one, so $f = \frac{1}{T}$ and $T = \frac{1}{f}$.

25.3 Wave Motion

A wave is a mechanism to transfer energy without transferring mass. You can think of a wave as an oscillation that moves from one location to another such as what happens to the surface of a pond when you throw a stone into it — the ripples move outward in all directions from the initial disturbance.

25.4 Wave Speed

The speed of a wave (v) depends on the medium through which the wave moves. This course will not get into the specifics of how to determine wave speed but you should know that the more dense the medium the slower the speed will be and the higher the pressure or tension the higher the speed will be.

Since the frequency of a wave is the number of full waves that pass a point in one second and the wavelength is how long each of those waves is the speed of the wave is given by $v = f\lambda$.

25.5 Transverse Waves

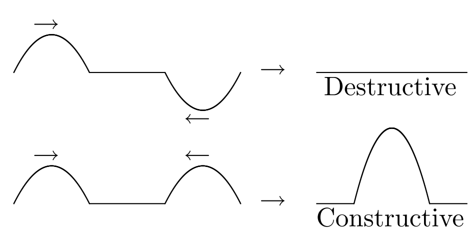
Transverse waves are waves that vibrate perpendicular to the direction of wave travel like a wave in a stretched string.

25.6 Longitudinal Waves

Longitudinal waves or compression waves are waves that vibrate forward and backward along the direction of travel like sound waves traveling through air.

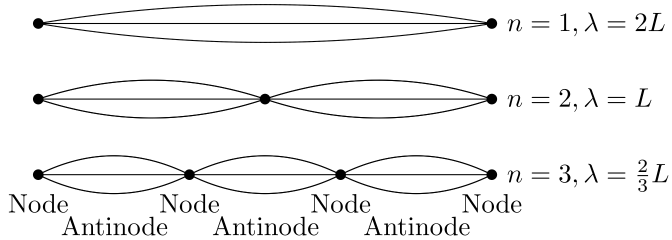
25.7 Interference

Unlike two physical objects, two waves are able to move through each other. Think about the result of dropping two stones into the water a small distance apart, the waves coming out from each of them will overlap and form an **interference pattern.** If two waves line up so that a crest in one wave is on top of a crest in the other then the result will be what is known as **constructive interference** and the resulting wave will have a larger amplitude than either of the two that are overlapping. If a crest lines up with a trough of the other wave then the result will be **destructive interference** and the resulting wave will be smaller, possibly zero. If two waves arrive at a point where the are in step with each other we say that the waves are **in phase** with each other, if they are out of step and cancel each other then we say that they are **out of phase** with each other.



25.8 Standing Waves

When a wave travels along through some medium and comes to a more dense (or rigid) medium then it will reflect back off of that boundary. The incident wave will form an interference pattern with the reflected wave, and if the wavelength of the wave is just right then what is known as a **standing wave** will be formed where the entire medium will appear to vibrate together. There are multiple wavelengths that will form a standing wave in the medium, the longest three wavelengths of them will produce patterns like this:



In the bottom diagram the four points where there is no amplitude to the vibration are labeled as **nodes** and the points in between them where the vibration has maximum amplitude are marked as **antinodes.**

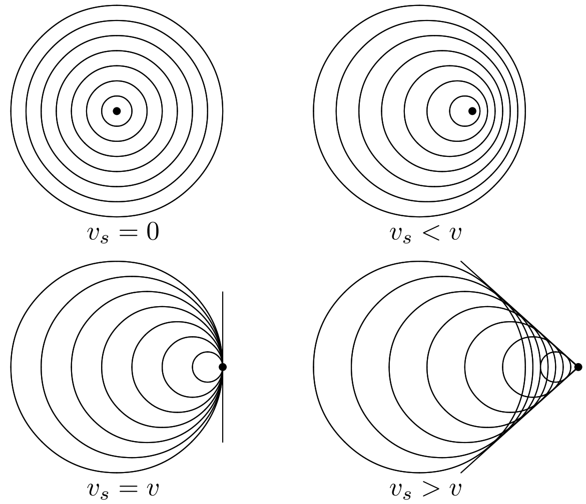
n	Nodes	Antinodes	λ
1	2	1	$\lambda = 2L$
2	3	2	$\lambda = L$
3	4	3	$\lambda = \frac{2}{3}L$
4	5	4	$\lambda = \frac{1}{2}L$
5	6	5	$\lambda = \frac{2}{5}L$
		...	
n	$n+1$	n	$\lambda = \frac{2}{n}L$

25.9 The Doppler Effect

If the source and detector of a sound are moving toward or away from each other the frequency of the sound observed will change depending on the motion. If they move toward each other the frequency observed will be higher than than they are at rest, if they move away from each other then the observed frequency will be lower. This is known as the **Doppler effect.**

25.10 Bow Waves

If the source of a wave moves as fast as or faster than the speed of the wave then something different will be observed. Consider a boat causing ripples as it moves across a calm lake with some speed v_s.



In the case where the source is not moving ($v_s = 0$) a stationary observer at a will see the ripples pass their location at the same frequency as they were emitted. If the source is moving slowly ($v_s < v_w$) a stationary observer at b will see the ripples pass their location at a lower frequency than when they were emitted and one at c will see the ripples pass at a higher frequency than when they were emitted.

In the case where the source is moving at a speed equal to the wave speed ($v_s = v_w$) then the waves will move forward along with the front of the boat, none will pass it, and the fronts of the waves will move forward in a straight line. If the source is moving at a speed greater than the wave speed ($v_s > v_w$) then the boat will pass the fronts of the previously emitted waves and the fronts of the waves will move outward in a 'v' shape that is commonly described as a wake.

25.11 Shock Waves

The three-dimensional version of a bow wave comes from a very fast plane moving through the air. Rather than a flat 'v' on the surface of a lake a plane moving faster than the speed of sound will produce a **shock wave** that follows behind the plane in a cone.

Chapter 26 — Sound

26.1 The Origin of Sound

All sounds are produced by the vibration of some object. In a musical instrument that is usually either a vibrating string or reed or perhaps a moving column of air in part of the instrument. The production of sound is not limited to musical instruments, any vibrating object will cause similar vibrations in the air.

If the vibrations are between 20 Hz and 20 000 Hz then they are in the range that can be typically heard by a person with normal hearing. Any waves below that range (less than 20 Hz) are called **infrasonic**, those above that range (greater than 20 000 Hz) are called **ultrasonic**. Humans cannot hear infrasonic or ultrasonic waves. The frequency that we perceive for those waves we do hear is called the **pitch** of the wave.

26.2 Sound in Air

The vibrating object that is the source of a sound will create a longitudinal wave composed of successive **compressions** (higher-pressure areas, drawn as the crests of a wave) and **rarefactions** (lower-pressure areas, drawn as the troughs) that will travel outward from the source.

26.3 Media That Transmit Sound

We are used to sound traveling through the air but it will also travel through many other materials. In fact, sound will travel better and faster through solid objects and liquids than through the air.

Like any other mechanical wave, sound cannot travel in absence of a medium. A vacuum will prevent sound from passing since it contains nothing to vibrate.

26.4 Speed of Sound

Light travels much faster than sound, for this reason you will see a distant event (such as an explosion or lightning strike) before you hear the accompanying sound. Light travels fast enough that for most purposes its speed can be ignored and it can be assumed to travel instantly from one place to another.

The exact speed that sound waves travel in air varies with the temperature, barometric pressure, and humidity, about 340 meters per second at room temperature is typical. They travel faster in warmer air and slower in cooler air.

In a more elastic material such as a solid or a liquid the atoms are packed more closely together and have a stronger force restoring them to their original position once they are disturbed, this causes sound to travel much faster through those materials. In water a speed of 1500 meters per second is typical, in steel it is close to 5100 meters per second. The exact values will depend on the alloy, chemical impurities and temperature of the material.

26.5 Loudness

The intensity of a sound wave is proportional to the square of its amplitude. The loudness of a sound will increase with the intensity but it will not be a proportional relationship, instead it will increase with the logarithm of the intensity. Each time the the loudness seems to double the intensity will have increased by a factor of ten.

The units used to measure intensity are **decibels** (dB), an intensity of 0 dB corresponds to the **threshold of hearing** or the quietest sound that a person can typically hear. Every increase of 10 dB will cause the intensity to increase by a factor of 10, increasing by 20 dB will increase the intensity by a factor of 100.

At the same time, since the loudness that you will perceive doubles each time the intensity increases by a factor of 10 the loudness will double with each 10 dB increase, a 20 dB increase will cause the sound to be four times as loud.

26.6 Forced Vibration

If a small object is vibrating then a soft sound will be heard, if the same object is placed against a larger flat surface then the larger surface act as a sounding board and will cause more air to vibrate so a louder sound will result. This phenomenon is known as **forced vibration**.

26.7 Natural Frequency

When an object that is made of an elastic material is struck it will vibrate at a frequency (or a set of frequencies) that remains constant for that object. This is the object's **natural frquency**. Consider a bell, if a bell is rung it will always sound the same, a different bell may have a different sound but it will also always be the same for that bell.

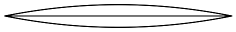
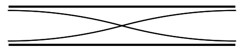
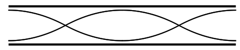
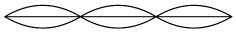
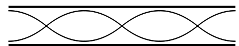
26.8 Resonance

If the frequency of forced vibration matches an object's natural frequency then the resulting sound will increase in intentity and loundness, this is called **resonance**. If you recall the lab that we did regarding the speed of sound you were looking for the length of an air column where resonance would occur, if it was too long or too short the sound would be barely louder than the tuning fork but when the resonant length was found it would be much louder.

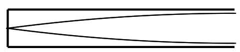
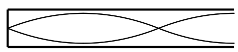
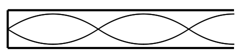
26.9 Interference

Since sound wave share the properties of any other wave hey also show the effects of interference when two or more waves reach the same location. If they arrive in phase the result will be **constructive interference**, meaning that the re-

Standing Waves on a String or Open Pipe of Length L

n	String Diagram	Pipe Diagram	Wavelength	Frequency
1			$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$
2			$\lambda_2 = L$	$f_2 = \frac{v}{L} = 2f_1$
3			$\lambda_3 = \frac{2}{3}L$	$f_3 = \frac{3v}{2L} = 3f_1$
n	$n = 1, 2, 3, \ldots$		$\lambda_n = \frac{2}{n}L$	$f_n = n\frac{v}{2L} = nf_1$

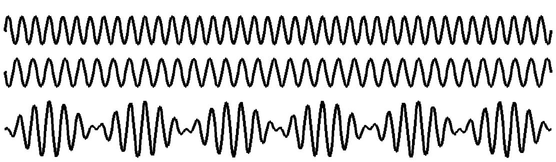
Standing Waves in a Pipe Closed at One End

n	Diagram	Wavelength	Frequency
1		$\lambda_1 = 4L$	$f_1 = \frac{v}{4L}$
3		$\lambda_3 = \frac{4}{3}L$	$f_3 = \frac{3v}{4L} = 3f_1$
5		$\lambda_5 = \frac{4}{5}L$	$f_5 = \frac{5v}{4L} = 5f_1$
n	$n = 1, 3, 5, \ldots$	$\lambda_n = \frac{4}{n}L$	$f_n = n\frac{v}{4L} = nf_1$

sulting wavefor will be larger than either of the waves that arrived. If the waves arrive out of phase then the **destructive interference** that occurs will case the resulting wave to be smaller than either of the waves that are interfering, perhaps even completely cancelled out. See the diagram at the top of page 397 in your textbook for examples of constructive and destructive interference.

26.10 Beats

If two sources emit sound waves that are close to one another in frequency but not exactly the same then a **beat** will be observed. When the waves are added together the resulting sound oscillates between loud and soft. The number of beats heard per second is equal to the absolute value of the difference between the frequencies of the two sounds. The diagram below shows one second of the sum of a 34 Hz wave and a 40 Hz wave yielding a 6 Hz beat frequency.



$f = 40$ Hz
$f = 34$ Hz
Sum

Harmonics and Standing Waves

Strings and air-filled pipes can exhibit standing waves at certain frequencies. The lowest frequency able to form a standing wave is called the **fundamental frequency** (f_1), those above it are **harmonics** of that frequency. The frequency of each harmonic frequency is equal to the fundamental frequency multiplied by the harmonic number (n), this forms a **harmonic series**. The fundamental frequency is also referred to as the first harmonic frequency.

Depending on the medium in which the standing waves are formed the characteristics of the waves will vary. A pipe open at both ends has the same harmonic series as a vibrating string (producing all of the harmonics), while a pipe closed at one end will only produce odd-numbered harmonics.

The harmonics that are present form a sound source will affect the **timbre** or sound quality of the resulting sound.

Chapter 27 — Light

27.1 Early Concepts of Light

Thousands of years ago light was thought to originate from the viewer's eyes, later people learned that light originated from an external source and came to the viewer's eyes after interacting with other objects.

Once light was determined to have an external source there was still debate about whether light was a particle or a wave. In 1905 Einstein published a theory of how light was made up of particles of electromagnetic energy called **photons**, later it was discovered that even though they are particles the photons still show characteristics of waves.

27.2 The Speed of Light

Originally light was thought to travel instantaneously from one point to another. In the second half of the seventeenth century light was found to have a very large but finite speed and the first attempts were made to find that speed.

The experiment that finally gave the value currently accepted as the speed of light was done by Albert Michaelson in 1880. He used a spinning octagonal mirror and a second mirror on a distant mountain to carefully determine the time that light took to travel that distance.

The speed of light in a vacuum is slightly faster than in air, that speed (c) has been found to be

$$c = 3 \times 10^8 \ \frac{\text{m}}{\text{s}} = 300\ 000 \ \frac{\text{km}}{\text{s}}$$

Light takes about 8 minutes to travel from the sun to the earth.

The distance that light travels in one year is called a **light-year** and is used to measure very long distances such as between stars. The closest star to the sun, Alpha Centauri, is about 4 light-years away.

27.3 Electromagnetic Waves

The energy emitted by vibrating electric charges is partly electric and partly magnetic so it is referred to as an **electromagnetic wave**. Light is a small part of the **electromagnetic spectrum** with wavelengths from 400 nm to 600 nm. Longer wavelengths are **infrared** (below red), microwaves, and radio waves. Shorter wavelengths are **ultraviolet** (above violet), x-rays, and gamma rays. There is a diagram in your textbook on page 408 that shows the relationship between the parts of the electromagnetic spectrum.

27.4 Light and Transparent Materials

When light travels through a **transparent** material it is successively absorbed and re-emitted by the atoms that it strikes. This takes longer than the light would take to travel the same distance with no medium in the way, therefore light is slowed in a transparent medium. Some media will pass electromagnetic waves of one frequency while blocking those of another frequency, an example would be ordinary window glass which passes visible light while absorbing or reflecting ultraviolet.

27.5 Opaque Materials

Materials which absorb light without re-emitting it are called **opaque**. When light shines onto an opaque object the energy will be transformed into heat, warming it.

27.6 Shadows

A thin beam of light as we will work with it later is called a **ray**. A broader beam of light can be thought of as a collection of rays, perhaps all shining in the same direction, perhaps shining in different directions. Anything that those rays would have reached if they were not blocked by some intervening object is said to be in that object's **shadow**. If all of the rays are blocked from reaching a part of the surface then it is in the **umbra** of the object, if only some of the rays are blocked then it is in the **penumbra**. Your textbook has diagrams and an explanation of how this applies to eclipses on page 413.

27.7 Polarization

Light can be caused to vibrate in one direction through **polarization**. Light traveling forward will normally vibrate up and down, left and right, and everything in between. When it is polarized it will all vibrate in a single direction. If the polarized light encounters a second polarizing filter only the component of the originally polarized light that is in the direction of the second filter will pass through. If the directions match then nearly all of the light will pass through, if the directions are at right angles to each other then nearly all of the light will be blocked.

Light can also be polarized by reflecting off of a surface, if light bounces off of a horizontal surface at a low angle it will become horizontally polarized.

27.8 Polarized Light and 3-D Viewing

Polarized light is used to produce 3-D images. Two projectors are used at one time, one projector has a horizontal polarizing filter over the lens, the second one has a vertical polarizing filter. When the result is viewed through glasses that have a similar combination of polarizing filters each eye will see the view from one of the projectors. If the images are set up correctly then the viewer will see a 3-D image.

Chapter 28 — Color

28.1 The Color Spectrum

If you shine a beam of **white light** through a triangular prism you can see the light bend and split up by color into a **spectrum**. Red light will be bent least, violet will be bent most, and in between the order will be red, orange, yellow, green, blue, and violet.

If all of the colors of the spectrum were to be recombined the result will be white light again. If all of the light is removed the result will be black.

28.2 Color By Reflection

When white light shines on an object some of the light will be absorbed and some will be reflected. Depending on the material the object is made of some wavelengths will be absorbed more than others, the remaining ones will be reflected. The colors of light that are reflected will be the ones that you see when you look at the object, they will determine the color that you see.

You can only see the colors reflect from an object that shine on the object to begin with, if you shine a single color of light on an object, for example red, if the object reflects red light it will appear red, if not then it will be black. No other colors are possible besides the ones shining on the object.

28.3 Color By Transmission

Similar to how any object will absorb some wavelengths or light while reflecting others, if white light shines through a transparent object some wavelengths will be absorbed while others will pass through. The energy carried colors absorbed by the object will be turned into heat and will cause its temperature to slightly increase.

28.4 Sunlight

Light from the sun is a composite of all of the visible frequencies as well as many others that we cannot see. The brightest colors coming from the sun are yellow and green with slightly less intensity as the frequencies move away from that point. Because humans developed in that light we are most sensitive to those frequencies. This increased sensitivity is why many emergency vehicles such as fire engines are being painted that color.

28.5 Mixing Colored Light

The primary colors of light (the **additive primary colors** are red, blue, and green. All three together produce white light, red and green produce yellow, red and blue produce magenta, and blue and green produce cyan. There are colored diagrams to illustrate this on pages 426 and 427 of your textbook.

28.6 Complimentary Colors

Complimentary colors are pairs of colors that together include each primary color once, for example cyan (blue and green light) is complimentary to red. When two complimentary colors of light are mixed the result is white light.

28.7 Mixing Colored Pigments

Pigments work by absorbing the complimentary colors to the ones that they reflect. A cyan pigment will absorb red light and reflect blue and green.

If two pigments are mixed the resulting pigment will absorb all colors absorbed by either of the pigments, this is known as mixing by subtraction. If yellow pigment (absorbs blue) is mixed with cyan pigment (absorbs red) the resulting pigment will absorb both blue and red and will reflect green light. A diagram of how this works and an example of a practical use of it is shown on pages 430 and 431 if your textbook. From the diagram and example you will see that the **subtractive primary colors** are cyan, magenta, and yellow.

28.8 Why the Sky Is Blue

As light passes through microscopic particles in the upper atmosphere some of the light will be absorbed and reemitted (**scattered**) by the particles it passes through. The shortest wavelengths of light (violet and blue) are scattered the most causing the sky to appear to be blue, the remaining colors of light (the longer wavelengths from green to red) will shine through causing the sun to appear to be yellow.

28.9 Why Sunsets Are Red

When the sun is low in the sky the sunlight has a much longer path through the atmosphere to reach your eyes. This means that more of the light (longer wavelengths) will be scattered leaving only red and orange light to reach the earth's surface.

28.10 Why Water Is Greenish Blue

Water absorbs a very small amount of red light, when you see a small amount of water (a bottle or glass of water) you are unlikely to notice the coloration but when you see a large amount of water together the absence of red makes the water look greenish blue or cyan.

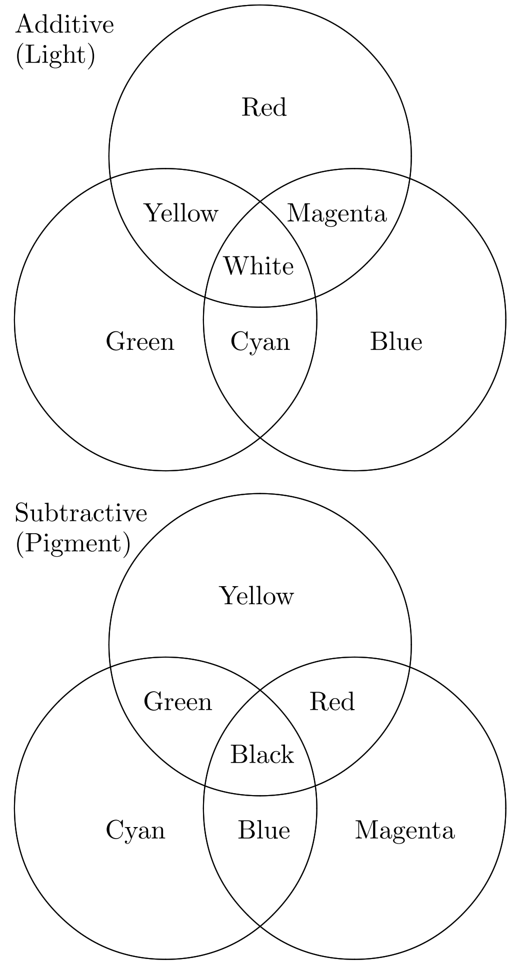
28.11 The Atomic Color Code — Atomic Spectra

Every element emits its own characteristic set of colors of light when it is heated in a gaseous state. If the light coming from some element is viewed through a **spectroscope** then a pattern known as a **line spectrum** will be seen that is different for every element. Several examples of line spectra are shown on page 438 of your textbook.

Primary Colors and Color Mixing

Additive and subtractive primary colors

Color	Additive (mixing light)	Subtractive (mixing pigment)
red	primary	complimentary to cyan
green	primary	complimentary to magenta
blue	primary	complimentary to yellow
cyan (blue green)	complimentary to red	primary
magenta (red blue)	complimentary to green	primary
yellow	complimentary to blue	primary



Chapter 29 — Reflection and Refraction

29.1 Reflection

When a wave reached the boundary between two media some or all of the wave will **reflect** back into the original medium. If all of the wave energy is reflected then it is said to be **totally reflected**, if some of it passes into the second medium then it is said to be **partially reflected**.

29.2 Reflection

If a wave is traveling in a one-dimensional medium (such as a pulse traveling along a spring) then the wave can only go forward and backward. When two or three dimensions are involved (we will stick to only two in this class) waves will reflect off of a boundary the same way a ball would bounce off of a wall — they will bounce off of the boundary at the same angle they arrived but on the other side of an imaginary perpendicular line called the **normal line**. The angles that the wave arrives and departs at are measured from the normal line, the angle it arrives at is called the **angle of incidence** (θ_i) and the departing angle is the **angle of reflection** (θ'). This leads to the mathematical relationship known as the **law of reflection**:

$$angle\ of\ incidence = angle\ of\ reflection$$

or

$$\theta_i = \theta'$$

29.3 Mirrors

If you look at an object in a plane mirror (an ordinary flat mirror) you will see the object some distance behind the mirror. The rays of light reaching your eye do not actually start at the position of the image of the object so the image is not actually there, but your eye can't tell the difference when looking in the mirror. An image like this that you can see but isn't actually there is called a **virtual image**.

The virtual image in a plane mirror is always the same size as the object itself and will seem to be the same distance behind the mirror as the object is in front of it.

29.4 Diffuse Reflection

A mirror is by design a very smooth surface but light can reflect from other surfaces as well. When light bounces off of a rough surface the law of reflection is still obeyed but consider what happens if you try to catch a ball that's going to bounce off of rough ground — depending on exactly where it strikes the ground the surface it hits could be tilted in any number of directions making the final direction depend on exactly where it strikes, a small difference in position could make a large difference in the direction. The same thing happens when light strikes a rough surface — the light ray will reflect perpendicular to the surface where it strikes but not all of the surface is facing the same direction. This results in **diffuse reflection**.

What is considered rough vs. smooth depends on the wavelength of the wave striking the surface, if the bumps on the surface are small compared to the wavelength of the wave then it's a smooth surface, if they're large then it's a rough surface.

29.5 Reflection of Sound

Sound is a wave just like light or radio, the main difference is that sound has a much longer wavelength. When sound bounces back off of a distant object with a noticeable delay it's called an **echo**, if it bounces back and forth in a room it's often called **reverberation**. Rooms like concert halls are designed to reduce unwanted reverberation to improve the quality of the sound for the audience.

29.6 Refraction

If a wave strikes a boundary between two different media at an angle then the wave will be bent or **refracted** as it passes into the second medium. This is because the speed of the wave changes and the side of the wave that strikes the slower medium first (or leaves it last) will drag a little and cause the wave to turn in that direction. The exact amount that the wave will turn depends on the relative speeds of sound in the two materials.

If you draw lines along the fronts of waves as they travel it can be helpful to visualize them, these are called **wave fronts** and will help to show how a wave changes its motion when changing media. See page 449 of your textbook for examples.

29.7 Refraction of Sound

Waves do not need a hard boundary between media in order to refract, they will also bend if there is a gradual change in the speed such as in the atmosphere when a layer of cool air is under a layer of warm air, the sound moving faster in the warm air will cause it to curve downward a bit causing distant sounds to seem to carry farther than they normally would.

29.8 Refraction of Light

When light rays move from a medium where light travels quickly such as air to one where it travels slower such as water the light rays will refract toward the normal line, if they go in the opposite direction then they will refract away from the normal line.

More precise results require the use of mathematics.

The index of refraction (n) of a medium is defined to be:

$$index\ of\ refraction = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ the\ medium}$$

For an index of refraction in an incident medium n_i and an index of refraction in a refracted medium n_r the relationship between the angle of incidence (θ_i) and the angle of refraction (θ_r) is given by **Snell's law**:

$$n_i \sin\theta_i = n_r \sin\theta_r$$

29.9 Atmospheric Refraction

Just as sound waves will gradually bend when there is a small change in its speed in the air because of temperature changes light waves can be observed to do the same thing. If the air right above the ground is warm but the air above that is cooler then light waves will curve upward somewhat. If you look into the distance this can cause you to see a **mirage** where you see what appears to be a reflecting surface such as water on the ground some distance away.

A similar effect will cause you to see the sun just before it actually rises over the horizon and just after it sets, this makes a difference of about 5 minutes in the length of every day.

29.10 Dispersion in a Prism

Various factors cause blue and other short-wavelength light to travel through some media (such as glass and plastic) at different speeds than longer-wavelength light such as red. Since all colors of light travel at nearly the same speed in air (and exactly the same speed in a vacuum) this means that the index of refraction for some materials varies for different colors of light. The colors with higher indices of refraction (the blue end of the spectrum) will refract a bit more than the ones with lower indices of refraction (the red end). The result of this will be a visible spectrum being produced when light shines through a prism.

29.11 The Rainbow

Just as a spectrum is produced when light shines through a prism it can also be produced when light shines through other transparent media such as raindrops. Your textbook has an illustration on pages 455–466 of how this will form the rainbow you are familiar with.

29.12 Total Internal Reflection

Since light traveling from a dense medium to a less dense one will speed up and refract away from the normal line there will eventually be some angle of incidence where the angle of reflection is 90° or greater. The angle of incidence that makes the angle of refraction exactly 90° is called the **critical angle**. Your textbook illustrates this on page 458.

Chapter 30 — Lenses

30.1 Converging and Diverging Lenses

A **lens** is a piece of glass or other transparent material that is specially shaped to change the path of light passing through it.

Lenses come in two types, **converging lenses** which cause parallel light rays to all come together to a single point (called the **focal point**) and **diverging lenses** which cause parallel light rays to spread out as if they came from a focal point on the other side of the lens. Converging lenses are thin at the edges and thicker in the middle, diverging lenses have the opposite shape.

When drawing a diagram of an optical system start with a **principal axis** drawn perpendicular to and through the center of the lens. The focal points are marked on the principal axis one **focal length** from the lens. Your textbook illustrates this on page 464.

30.2 Image Formation by a Lens

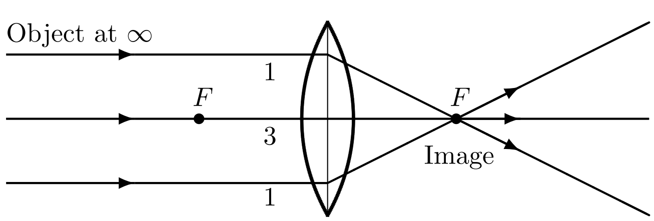
If the rays of light coming from an object actually come together to form the image it will be able to be projected on a screen so it is a **real image**, if it can't be projected because the rays do not actually converge at the image then it is a **virtual image**. Real images formed by a single lens are always upright, virtual ones are always inverted.

30.3 Constructing Images Through Ray Diagrams

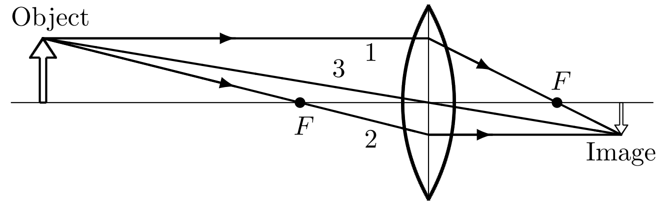
Ray diagrams can also be drawn to locate and describe the image that will be formed by a lens. The usual rays to be drawn are:

1. A ray that comes in parallel to the principal axis and goes out through the focal point.
2. A ray that comes in through the focal point and goes out parallel to the principal axis.
3. A ray that goes straight through the center of the lens

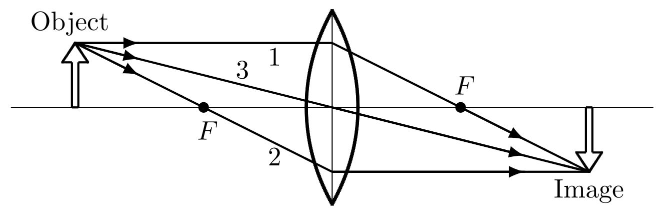
Each of these rays is labeled with its corresponding number in the ray diagrams below. We will use i to represent the **image distance**, the distance from the image to the lens, o to represent the **object distance**, the distance from the object to the lens, and f to represent the focal length of the lens. The focal points are labeled with F.



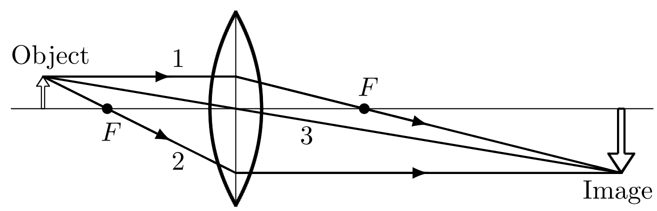
Converging lens, $p = \infty$, point image, $q = f$



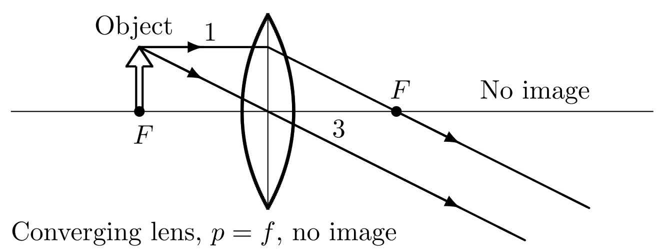
Converging lens, $p > 2f$, real image, $f < q < 2f$



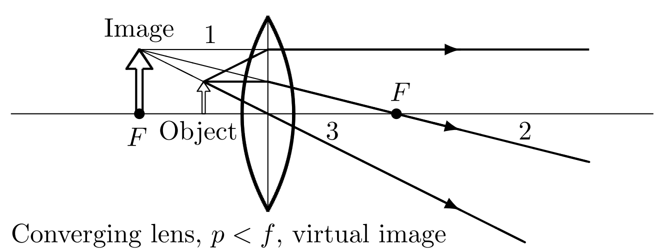
Converging lens, $p = 2f$, real image, $q = 2f$



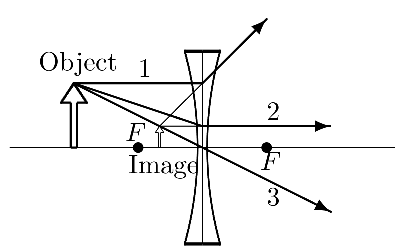
Converging lens, $f < p < 2f$, real image, $q > 2f$



Converging lens, $p = f$, no image



Converging lens, $p < f$, virtual image



Diverging lens, virtual image

The relationship between the focal length (f), object distance (p), and image distance (q) can also be found from the **thin lens equation**:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

30.4 Image Formation Summarized

Any image formed by a diverging lens will be upright and virtual.

An image formed by a converging lens will be inverted and real if the object is farther than the focal length from the lens, will be upright and virtual if the object is between the lens and the focal point, and no image will be formed if the image is at the focal point.

30.5 Some Common Optical Instruments

Pages 471–473 of your textbook illustrate how lenses can be used to construct a camera, a telescope, a microscope, and a projector.

30.6 The Eye

Pages 473 and 474 of your textbook discuss the structure of your eye. Of interest for this class is the converging lens at the front of your eye which can change its focal length as needed (within a certain range) and the retina at the back of the eye. The lens will change focal length to bring an image of whatever is being looked at into focus on the retina.

30.7 Some Defects in Vision

Instead of forming images on the retina as they are supposed to, some eyes will form an image behind the retina (**farsighted**) or in front of it (**nearsighted**). These can be corrected by placing corrective lenses in front of the eyes, either a converging lens to correct farsightedness or a diverging lens to correct nearsightedness.

30.8 Some Defects of Lenses

Lenses have defects called **aberrations**. The most common are **spherical aberration** (light passing through the edges of a spherical lens will focus at a different point than light passing close to the center) and **chromatic aberration** (light of different colors will focus at different points.

Correcting spherical aberration requires that lenses be made parabolic in cross section rather than as a section of a sphere, this is a considerably more expensive process.

To correct chromatic aberration requires that carefully chosen pairs (or more) of lenses be used so that their chromatic aberrations will cancel each other out.

Chapter 31 — Diffraction and Interference

31.1 Huygens' Principle

Christian Huygens developed what is now known as **Huygens' principle** which states that every point on a wave front acts as the source of a new wavelet that spreads out in a sphere from that point. This is illustrated on pages 481 and 482 of your textbook.

31.2 Diffraction

If a wave bends because of anything other than reflection or refraction it is called **diffraction**. Whenever waves pass through an opening or past an edge they will spread, the degree to which they spread will depend on the size of the opening and the wavelength of the waves.

When light passes through a large window very little diffraction will be seen, if it passes through a small slit then the diffraction will be much more pronounced. For a fixed opening the longer the wavelength of the wave the more diffraction that will be observed. Long waves such as radio waves (especially AM radio) will diffract much more than shorter waves like microwaves and visible light.

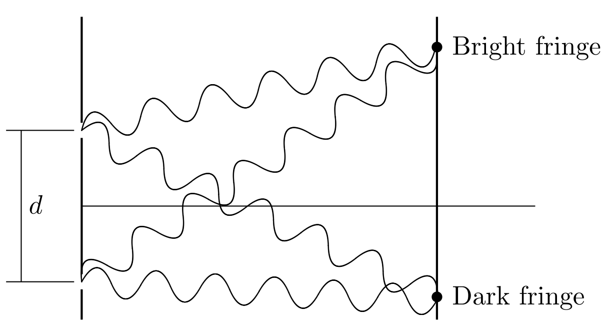
Diffraction also limits the ability of microscopes and other optical instruments to see small objects, once the size of the object is comparable to the wavelength of the light it will no longer produce a clear image.

31.3 Interference

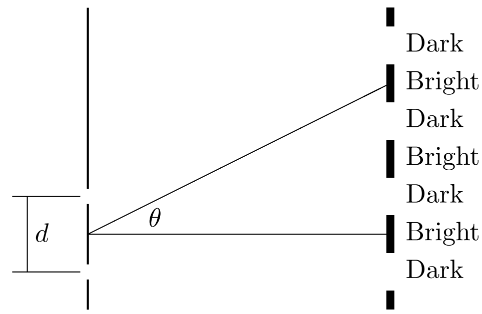
If any two waves arrive in phase with each other they will produce a larger amplitude through constructive interference, if they are out of phase then they will cancel each other (in whole or in part) through destructive interference.

31.4 Young's Interference Experiment

In 1801 Thomas Young discovered that **monochromatic** light (light of all one wavelength) created an interference pattern when it was allowed to shine through two small closely-spaced slits. The waves coming from those slits would arrive either in or out of phase at a screen, and produce either a bright or dark fringe depending on that phase.



The pattern will keep repeating, alternating bright and dark fringes. Each fringe will be located at the same angle from a perpendicular line regardless of the distance to the screen.



A similar pattern is observed if the light instead shines on a **diffraction grating** (a glass or plastic plate with a large number of regularly-spaced grooves or slits). The pattern will be the same as if the light struck a single pair of grooves on the diffraction grating.

31.5 Single-Color Interference from Thin Films

When monochromatic light reflects off of two surfaces that are almost (but not quite) parallel to each other a different set of interference fringes will result. As shown on page 490 of your textbook, depending on the thickness at that point either constructive or destructive interference will result.

31.6 Iridescence of Thin Films

The position of thin-film interference fringes depends on the wavelength of the light illuminating the objects, as the wavelength of the light increases so does the width of and spacing between the fringes. As the various colors come into constructive and destructive interference the resulting colors observed at each point will be a combination of all of the colors experiencing constructive interference there. As an example consider the bands of color you would observe looking at a soap bubble or at gasoline spilled on a wet surface.

31.7 Laser Light

A beam of light with the same frequency (and thus wavelength), phase, and direction is said to be **coherent**. There is no destructive interference between the individual waves in a beam of coherent light.

A common source of coherent light is a **laser** (originally an acronym for "Light Amplification by Stimulated Emission of Radiation"). A laser does not create energy, it just delivers its energy output in a way that makes it seem to be very bright. A laser commonly only outputs less than 1% of the energy put in to it.

31.8 The Hologram

A **hologram** is a three-dimensional picture of an object that contains an interference pattern generated by spreading a laser beam so that half of it shines on the object and the other half shines on a piece of film. When the two beams combine on the film they create an interference pattern. If a similar beam of light shines on the developed hologram then the waves will recreate the original waves that formed the interference pattern and the result will be a three-dimensional view of the original object.

Chapter 32 — Electrostatics

32.1 Electrical Forces and Charges

Pairs of electrical charges have a force that acts between them similar to the way that pairs of masses have a force acting between them. Unlike gravity which can only be an attractive force the **electrical force** can be either attractive or repulsive. Two charges with opposite signs (one positive, one negative) will attract each other, two with the same sign (either both positive or both negative) will repel each other.

32.2 Conservation of Charge

The principle of **conservation of charge** states that the total amount of charge, both positive and negative, is a constant. Charge can be transferred from one object to another but it cannot be created or destroyed. If one object is given a positive charge something else must be given a negative charge of the same magnitude.

32.3 Coulomb's Law

The electric force between two charges is given by **Coulomb's law** which states:

$$F = k_C \frac{q_1 q_2}{d^2}$$

This is dependent on the Coulomb constant (k_C):

$$k_C = 9.0 \times 10^9 \ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

The direction of the force will be toward the other charge if the two charges have opposite signs or away from the other charge if they both have the same sign.

32.4 Conductors and Insulators

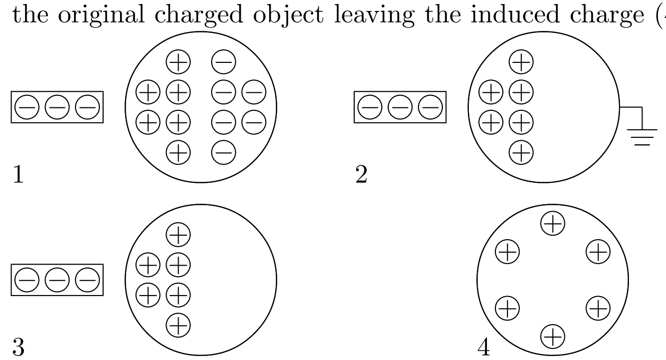
Some materials will allow electrons to freely move through them, others prevent the easy movement of electrons. If electrons are easy to move in a material that material is called a **conductor**, if the electrons are tightly bound to individual atoms and do not easily move the material is called an **insulator**. Metals generally make very good conductors. Examples of insulators include plastic, rubber, and wood.

32.5 Charging by Friction and Contact

We can cause an object to become electrically charged though several methods, the one you are probably most familiar with is charging by friction (or contact). When two dissimilar materials contact each other some of the electrons in one material will be drawn into the other material, this effect can be increased by rubbing the two surfaces together.

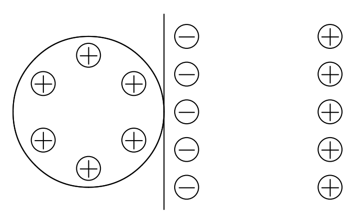
32.6 Charging by Induction

If a charged object is already available then it can be used to charge another object by **induction**, even without touching the second object. This is done by bringing another charged object such as a rubber rod near the object to be charged (1, below), allowing the displaced charge to drain off on the other side (2) then removing the drain while the charged object remains (3) then finally removing the original charged object leaving the induced charge (4).



32.7 Charge Polarization

When an insulator is brought near a charged object the charges in the insulator will rearrange slightly so that one side will have a slight positive charge and the other will have a slight negative charge, causing the object to become **electrically polarized**. This will not change the net charge of the insulator but it will be enough to allow a small or light object to be lifted by a charged object.



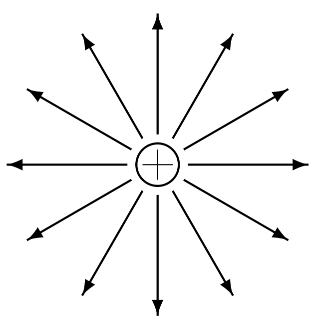
Chapter 33 — Electric Fields and Potential

33.1 Electric Fields

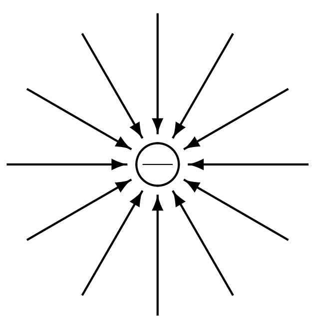
Just as a gravitational field exists around any object with mass that will exert a force on any other mass in that field, there is an **electric field** around any charged object that will exert a force on any other charged object placed in that field.

33.2 Electric Field Lines

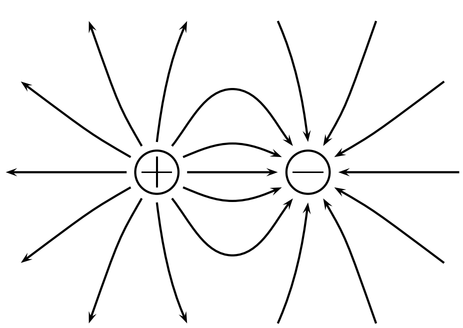
An electric field around one or more charges can be represented by drawing lines to show the strength and direction of the field. They are illustrated on pages 519–520 of your textbook. When drawing field lines there are three main rules to follow: 1) Field lines must begin on positive charges or at infinity and must terminate on negative charges or at infinity; 2) The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge; and 3) No two field lines from the same field can cross each other.



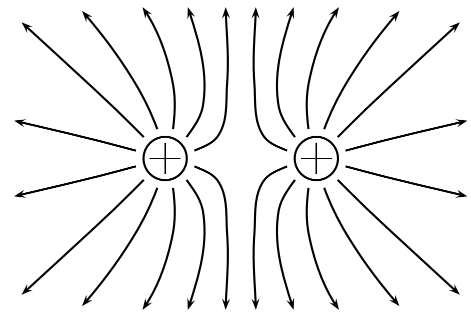
A single positive charge



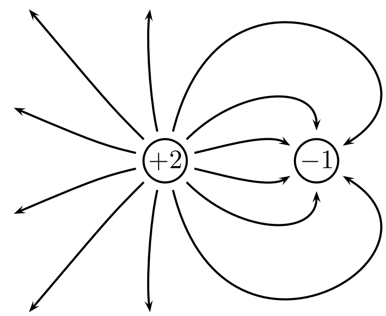
A single negative charge



Equal positive and negative charges



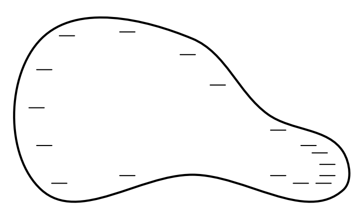
Equal positive charges



A postive charge twice the size of a negative charge

33.3 Electric Shielding

Static charges on a conductor distribute themselves to be as far from each other as possible. This will cause the charges to accumulate on the outside surface of the conductor with none of the unbalanced charges in the center. This will also cause the largest accumulation of charge to happen where the radius of curvature is smallest, the more pointed a part of the object is the more of the charge will accumulate there as shown below:



As a result of the charge moving to the outer surface there will be no electric field on the inside of a conductor.

33.4 Electric Potential Energy

Potential energy exists anywhere work has been done to put an object into a new position (assuming that the energy has not been lost to friction). Just as it takes work to lift an object against Earth's gravity it also takes work to move an object in an electric field. The work done to move the object in the electric field will be stored in the object as **electric potential energy** and can be recovered by allowing the object to return to its initial position.

33.5 Electric Potential

Finding electric potential energy requires that the charge on the object to be moved be known, at times it is useful to be able to find the amount of electric potential energy an object would have without knowing in advance the object's charge. This leads to the idea of finding the electric potential energy per unit charge for some location in an electric field which is known as **electric potential** and is defined as:

$$electric\ potential = \frac{electric\ potential\ energy}{charge}$$

Electric potential is measured in joules of energy per coulomb of charge, a quantity known as a **volt** (V). Because of the unites used to measure it electric potential is often known as **voltage**.

33.6 Electric Energy Storage

Electrical energy can be stored in devices known as **capacitors**. A capacitor contains two plates or strips, one of which will take a positive charge and the other a negative. Placing the two plates close together with an insulator between them will cause the opposite charges to hold each other in place until some way is given for the charges to equalize, usually by connecting an electric circuit between the plates.

33.7 The Van de Graaff Generator

A **Van de Graaff generator** is a device for building a very high voltage charge on a metal sphere which can then be used for various purposes. A diagram of one is in your textbook on page 527. It works by causing a small static charge to be deposited on a moving belt in the base, the belt carries the charge to the top sphere where it is removed from the belt on the inside of the sphere. The charge then travels to the surface of the sphere leaving the generator ready to move more charge off of the belt. This will continue until there is enough charge on the sphere to either cause a long spark to some other object or to ionize away into the air.

Chapter 34 — Electric Current

34.1 Flow of Charge

Whenever there is a **potential difference** between two points connected by a conductor some electric charge will flow through that conductor. The flow will continue as long as the potential difference is there. If the potential difference is because an object is charged then the flow will be fairly brief, if there's something to keep the potential difference going (such as a battery or a generator) then the flow can continue for a long time.

34.2 Electric Current

Electric current is the flow of electric charge. The charge is carried in a solid conductor by electrons that are able to move through the conductor, in gases and liquids then there may be positive ions also carrying some of the charge.

Current is measured by the amount of charge that flows past a certain point, one coulomb of charge flowing in one second is called an **ampere** (A or amp).

Wires carrying a current do not have a charge themselves, as electrons enter one end of the wire the same number leave the other end maintaining the wire's neutral charge.

34.3 Voltage Sources

Something that provides a potential difference between two points is known as a **voltage source**. Examples include batteries, generators, and anything else that will cause a continuous flow of charges. You can think of a voltage source as a pump that makes the electrons move through the conductor.

34.4 Electric Resistance

Electric resistance is the property of conductors to resist the flow of electric charges. Longer or thinner conductors have more resistance, shorter or thicker ones have less. The resistance also depends on the **conductivity** of the material, copper and aluminum have a high conductivity so wires made of those metals have a low resistance, other materials like nickel have a lower conductivity so they are only used in wires when a high resistance is needed.

Resistance is measured in **ohms** (Ω).

34.5 Ohm's Law

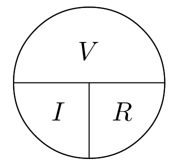
Ohm's law says that the current in a circuit (I) is equal to the voltage (V) divided by the resistance (R), so:

$$I = \frac{V}{R}$$

This can also be rearranged to solve it for R or V:

$$R = \frac{V}{I} \qquad V = IR$$

An easy way to remember it is to draw it like this:



If you put our finger on top of any one of the three quantities (V, I, or R) the remaining two will be in the right position to tell you what to do with them to find the one you have covered. For example, to find current cover the I with your finger, what you then see is $\frac{V}{R}$ which is equal to the current.

34.6 Ohm's Law and Electric Shock

When there is a potential difference across two parts of your body a small amount of charge will flow between them. Depending on the size of the current you may not notice it or you may be seriously injured.

If your skin is very dry it has a resistance of about 500 000 Ω but if it is soaked with salt water (or sweat) then that can drop to 100 Ω.

A current of about 0.001 A is the smallest that can be felt, about 0.005 A starts to be painful, and as little as 0.070 A can be fatal if it passes through your heart.

The power supplies that we use produce a small enough potential difference between their terminals that under normal circumstances they will not be able to produce enough of a current to be dangerous.

34.7 Direct Current and Alternating Current

Direct current (DC) is a flow of charge that is always in the same direction. Currents caused by charged objects or batteries are usually direct currents. **Alternating current** (AC) reverses direction many times per second. AC has several advantages for transmitting power over long distances and converting one voltage to another when needed so it is often used to deliver electricity to homes.

34.8 Converting AC to DC

A **diode** is a device that will allow a current in one direction but block it in another. Diodes can be used to pass only a part of an AC source to change it into a pulsing DC source, other circuit elements can be added that will smooth out the resulting DC output.

34.9 The Speed of Electrons in a Circuit

When a circuit is completed electricity begins to flow almost immediately, this does not mean that the electrons that leave the source make it through the circuit that quickly, only that they begin to move that fast. Individual electrons take much longer to make it around the circuit, moving at a **drift speed** of about 1 m every 3 hours.

34.10 The Source of Electrons in a Circuit

The electrons flowing through an electric circuit start out in the circuit itself. They are already present in the wires and other components, the potential difference causing the current just makes the electrons move, it does not supply them.

34.11 Electric Power

Electric power (P) is the electrical energy that is converted to heat as it flows through a circuit. The amount of power is the product of the current times the voltage ($power = current \times voltage$) and is measured in watts (W). Mathematically this is written

$$P = IV$$

Electrical energy from a utility company is billed in kilowatt-hours (kWh), one of which is the amount of energy consumed in 1 hour at a rate of 1 kilowatt or 3.6×10^6 J.

Chapter 35 — Electric Circuits

35.1 A Battery and a Bulb

An electric **circuit** forms a path between the two terminals of some potential difference. One of the simplest circuits is a battery and a light bulb connected between the terminals of the battery.

35.2 Electric Circuits

For electricity to flow there must be a continuous path for it, if it is interrupted (such as by opening a switch) then the current will stop. Multiple devices can be connected to the same circuit, if the electricity flows through one device after another then it is a **series** circuit, if the devices form multiple paths that the electricity can take then it is a **parallel** circuit.

35.3 Series Circuits

There are several characteristics of series circuits:

1. There is a single path through the circuit. If the current is interrupted at any point then all current will stop.
2. The total equivalent resistance of the circuit is the sum of the resistances of each element.
3. The current is equal to the potential difference divided by the total resistance. The current in each element is equal to the total current in the circuit.
4. The voltage drop across each element is equal to the current times the element's resistance.
5. The total voltage across the circuit is equal to the sum of the voltage drops across each element.

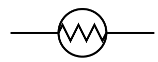
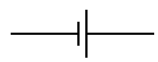
35.4 Parallel Circuits

There are several characteristics of parallel circuits:

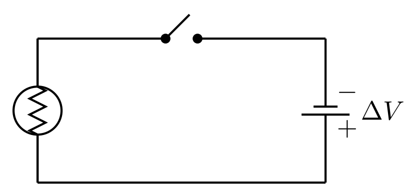
1. Each element of a circuit connects the same two points so the voltage across each element is the same.
2. The amount of current in each element is inversely proportional to the resistance of the element.
3. The total current in the circuit is equal to the sum of the currents through each element.
4. As the number of parallel branches increases the total resistance of the decreases. The total resistance will be less than the resistance of any individual element.

35.5 Schematic Diagrams

The common symbols used in electrical schematic diagrams are shown on page 554 of your textbook.

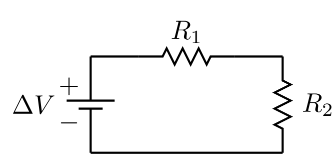
Component	Symbol	Comments
Wire		Wires are represented by drawing lines between the elements to be connected
Resistor or load		All resistive loads share this symbol. If a distinction needs to be made it is generally made by adding a label.
Lamp		A lamp (or bulb) is shown as a resistor with a circle around it to represent the glass
Battery		The longer line represents the positive terminal.
Switch		An open switch is shown. A closed switch would bring the arm down to make contact on both sides.

A schematic diagram of a simple flashlight could look like this



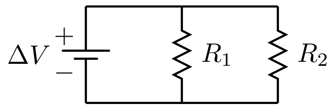
The battery is shown on the right, the lamp on the left, and a switch is at the top to allow the circuit to be opened or closed as desired. No details such as the flashlight housing, lens, or reflector are shown, they do not affect the electrical function of the flashlight so they are omitted.

35.6 Combining Resistors in a Compound Circuit



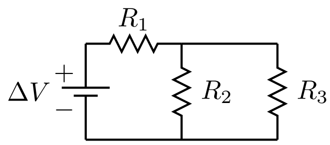
If two (or more) resistors are in series the total resistance is the sum of the individual resistances.

$$R_T = R_1 + R_2\ (+R_3 + \cdots)$$



If two (or more) resistors are in parallel then the inverse of the total resistance is the sum of the inverses of the individual resistances.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \left(+ \frac{1}{R_3} + \cdots \right)$$



If a circuit has resistors in both series and parallel then you just apply the same processes, starting with resistors that are only in series or only in parallel. Once the first few resistors are combined then the resulting equivalent resistors are combined with the remaining ones as shown on pages 555 and 556 of your textbook.

35.7 Parallel Circuits and Overloading

As more devices are added to a parallel circuit the total current will increase, possibly to more than the wires can safely handle. Fuses and circuit breakers are used to limit the total current in a circuit by opening the circuit if they see more than some preset limit.

Chapter 36 — Magnetism

36.1 Magnetic Poles

Just as electric charges produce electric forces **magnetic poles** produce magnetic forces. Each bit of a magnet has two poles, a **north-seeking pole** that wants to point north and a **south-seeking pole** that wants to point south. If you hang a straight magnet from a string you will see it rotate so that the north-seeking pole points north if there are not other strong magnetic fields nearby. Also, just as like electric charges repel and unlike ones attract, like magnetic poles repel and unlike ones attract.

Unlike electric charges which can be isolated (you can have a single positive or negative electric charge) magnetic poles always come in pairs. You cannot have a single north or south pole, only a set of one of each.

36.2 Magnetic Fields

A **magnetic field** is produced around any magnet and will affect other magnets placed nearby. If you sprinkle iron filings on and around a magnet you will see them line up along magnetic field lines. This is because the small bits of iron will act like miniature magnets when placed in the field and as such will line up along the stronger field.

36.3 The Nature of a Magnetic Field

In addition to the magnetic fields produced by magnets there are also magnetic fields produced any time electric charges are in motion. (In fact, the magnetic fields produced by magnets are also caused by moving charges as electrons spin in place, but we do not go into that level of detail in this class.)

36.4 Magnetic Domains

Materials such as iron have atoms that group together into clusters that all have the same magnetic orientation known as **magnetic domains**. In unmagnetized iron the domains are randomly oriented and mostly cancel each other out, as the iron starts to become magnetized the domains become aligned and start to constructively interfere. If the iron is strongly magnetized then almost all of the domains will have the same orientation.

Once an object is magnetized by aligning its magnetic domains if the object is then broken each part of it will have two equally strong poles.

36.5 Electric Currents and Magnetic Fields

Since moving charges create magnetic fields and currents are charges in motion it follows that electric currents will have magnetic fields around them as well. For a wire carrying current the magnetic field will wrap around the wire, if the wire is wrapped into a coil then the field will be strongest inside the coil. Photographs on page 569 of your textbook illustrate this.

36.6 Magnetic Forces on Moving Charged Particles

When a charged particle moves through a magnetic field in any direction other than parallel to the field the particle will feel a force from the magnetic field and be deflected. (The force is greatest when the particle moves perpendicular to the magnetic field lines.) This property is used in televisions to guide a beam of electrons to strike the correct colored dots on the screen.

36.7 Magnetic Forces on Current-Carrying Wires

If the moving charges are in a wire then they will transfer the force to the wire causing the wire to move. If they are strong enough they can evan cause a wire to jump out of a magnet.

36.8 Meters to Motors

Most (non-digital) electrical meters are made by wrapping a coil of wire around a magnet, when a current passes through the coil the magnet will experience a force and move causing a needle to move on a scale. By varying the circuit in which the meter is placed it can be set up to read current, voltage, or even resistance.

If the structure of a meter is enlarged then instead of just moving a needle the force on the magnet can be enough to produce a useful torque. With a few appropriate design changes this forms the basis for electric motors.

36.9 The Earth's Magnetic Field

The earth and everything on it make up a very large magnet. Near the north pole is the magnetic north pole (which is actually a south-seeking pole), when you are far from the pole the two are close enough together that the difference is of little concern (in this area the difference in direction between true north and magnetic north is about $8°$) but as you get closer to one of them it become more significant — there are several areas in the arctic where a magnetic compass will actually point south.

The exact cause of the earth's magnetic field is unknown, current theories lean toward movements in the molten part of the earth's interior and the charges that they carry with them as the cause.

Chapter 37 — Electromagnetic Induction

37.1 Electromagnetic Induction

When a magnet is moved in and out of (or through) a coil of wire it will induce a current in the coil though **electromagnetic induction**. Moving the magnet faster or increasing the number of turns in the coil will increase the current, moving the wire the other direction will reverse the direction of the current.

37.2 Faraday's Law

Electromagnetic induction is governed by **Faraday's law** which states *The induced voltage in a coil is proportional to the product of the number of loops and the rate at which the magnetic field changes within those loops.* The amount of work required to do this will depend on what the coil of wire is connected to.

37.3 Generators and Alternating Current

A **generator** is a device that continuously rotates a coil of wire inside a magnetic field, by Faraday's law as the amount of the magnetic field passing through the coil changes (or if you prefer as the coil cuts through magnetic field lines) a current will be induced in the coil and it can be drawn off through electrical contacts on the shaft of the generator. Work is required to rotate the generator, that work done is converted (after frictional losses) into electrical energy.

37.4 Motor and Generator Comparison

An electric motor and a generator are effectively the same device, the difference is where the energy is put in and where the energy comes out. A motor converts electrical energy to mechanical energy, a generator converts mechanical energy to electrical energy.

37.5 Transformers

If two coils of wire are placed near each other and one of the coils is passing a varying electric current then the second coil will have an induced current because of the changing magnetic field around the first coil. If the two coils are placed around the same iron core (as shown on page 584 of your textbook) this will create what is known as a **transformer**. A transformer can be used to increase or decrease the voltage coming from an alternating current source. The ratio by which the voltage will change is equal to the ratio of the number of turns on the output coil (the **secondary coil**) to the number of turns on the input coil (the **primary coil**). The total power handled by the transformer is the same in both coils. This can be expressed mathematically as

$$power = (voltage \times current)_{pri} = (voltage \times current)_{sec}$$

37.6 Power Transmission

Electric utilities supply electricity in the form of alternating current, this allows it to be stepped up to high voltages to send it long distances with little loss and then back down to 120 V to feed household circuits.

37.7 Induction of Electric and Magnetic Fields

Faraday's law can be generalized to the case where there may not be a conductor in a changing electric field, the new version states *An electric field is created in any region of space in which a magnetic field is changing with time. The magnitude of the created electric field is proportional to the rate at which the magnetic field changes. The direction of the created electric field is at right angles to the changing magnetic field.* If a charge is within the changing magnetic field it will experience a force from the induced electric field.

James Clark Maxwell promoted a counterpart to Faraday's law to predict the result of changing an electric field: *A magnetic field is created in any region of space in which an electric field is changing with time. The magnitude of the created magnetic field is proportional to the rate at which the electric field changes. The direction of the created magnetic field is at right angles to the changing electric field.*

37.8 Electromagnetic Waves

Any time an electric current is present it will generate a magnetic field, but the changing magnetic field around a changing current will also generate an electric field. If the two fields are arranged correctly the result will be an **electromagnetic wave** as shown on page 590 of your textbook. As discovered by Maxwell, electromagnetic waves (such as light, radio, x-rays, etc.) are changing electric and magnetic fields that move at exactly the same speed and reinforce each other. Maxwell showed that the only speed at which the waves can move is the speed of light.

Variables and Notation

SI Prefixes

Prefix	Mult.	Abb.	Prefix	Mult.	Abb.
yocto-	10^{-24}	y	yotta-	10^{24}	Y
zepto-	10^{-21}	z	zetta-	10^{21}	Z
atto-	10^{-18}	a	exa-	10^{18}	E
femto-	10^{-15}	f	peta-	10^{15}	P
pico-	10^{-12}	p	tera-	10^{12}	T
nano-	10^{-9}	n	giga-	10^{9}	G
micro-	10^{-6}	μ	mega-	10^{6}	M
milli-	10^{-3}	m	kilo-	10^{3}	k
centi-	10^{-2}	c	hecto-	10^{2}	h
deci-	10^{-1}	d	deka-	10^{1}	da

Notation

Notation	Description
$\vec{x}$	Vector
x	Scalar, or the magnitude of $\vec{x}$
$\lvert\vec{x}\rvert$	The absolute value or magnitude of $\vec{x}$
Δx	Change in x
$\sum x$	Sum of all x
Πx	Product of all x
x_i	Initial value of x
x_f	Final value of x
$\hat{x}$	Unit vector in the direction of x
$A \to B$	A implies B
$A \propto B$	A is proportional to B
$A \gg B$	A is much larger than B

Units

Symbol	Unit	Quantity	Composition
kg	kilogram	Mass	SI base unit
m	meter	Length	SI base unit
s	second	Time	SI base unit
A	ampere	Electric current	SI base unit
cd	candela	Luminous intensity	SI base unit
K	kelvin	Temperature	SI base unit
mol	mole	Amount	SI base unit
Ω	ohm	Resistance	$\frac{\mathrm{V}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A^2}}$
C	coulomb	Charge	$\mathrm{A \cdot s}$
F	farad	Capacitance	$\frac{\mathrm{C}}{\mathrm{V}}$ or $\frac{\mathrm{s^4 \cdot A^2}}{\mathrm{m^2 \cdot kg}}$
H	henry	Inductance	$\frac{\mathrm{V \cdot s}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{A^2 \cdot s^2}}$
Hz	hertz	Frequency	$\mathrm{s^{-1}}$
J	joule	Energy	$\mathrm{N \cdot m}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$
N	newton	Force	$\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$
rad	radian	Angle	$\frac{\mathrm{m}}{\mathrm{m}}$ or 1
T	tesla	Magnetic field	$\frac{\mathrm{N}}{\mathrm{A \cdot m}}$
V	volt	Electric potential	$\frac{\mathrm{J}}{\mathrm{C}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A}}$
W	watt	Power	$\frac{\mathrm{J}}{\mathrm{s}}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}$
Wb	weber	Magnetic flux	$\mathrm{V \cdot s}$ or $\frac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot A}}$

Greek Alphabet

Name	Maj.	Min.	Name	Maj.	Min.
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π or ϖ
Epsilon	E	ϵ or ε	Rho	P	ρ or ϱ
Zeta	Z	ζ	Sigma	Σ	σ or ς
Eta	H	η	Tau	T	τ
Theta	Θ	θ or ϑ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ or φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Variables

Variable	Description	Units
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ	Angular position	rad
θ_c	Critical angle	^{o} (degrees)
θ_i	Incident angle	^{o} (degrees)
θ_r	Refracted angle	^{o} (degrees)
θ'	Reflected angle	^{o} (degrees)
$\Delta\theta$	Angular displacement	rad
τ	Torque	$\text{N} \cdot \text{m}$
ω	Angular speed	$\frac{\text{rad}}{\text{s}}$
μ	Coefficient of friction	(unitless)
μ_k	Coefficient of kinetic friction	(unitless)
μ_s	Coefficient of static friction	(unitless)
$\vec{a}$	Acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_c$	Centripetal acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_g$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_t$	Tangential acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_x$	Acceleration in the x direction	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_y$	Acceleration in the y direction	$\frac{\text{m}}{\text{s}^2}$
A	Area	m^2
B	Magnetic field strength	T
C	Capacitance	F
$\vec{d}$	Displacement	m
$d\sin\theta$	lever arm	m
$\vec{d}_x$ or Δx	Displacement in the x direction	m
$\vec{d}_y$ or Δy	Displacement in the y direction	m
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
f	Focal length	m
$\vec{F}$	Force	N
$\vec{F}_c$	Centripetal force	N
$\vec{F}_{electric}$	Electrical force	N
$\vec{F}_g$	Gravitational force	N
$\vec{F}_k$	Kinetic frictional force	N
$\vec{F}_{magnetic}$	Magnetic force	N
$\vec{F}_n$	Normal force	N
$\vec{F}_s$	Static frictional force	N
$\vec{F}\Delta t$	Impulse	$\text{N} \cdot \text{s}$ or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$
$\vec{g}$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$

Variable	Description	Units
h	Object height	m
h'	Image height	m
I	Current	A
I	Moment of inertia	$\text{kg} \cdot \text{m}^2$
KE or K	Kinetic energy	J
KE_{rot}	Rotational kinetic energy	J
L	Angular Momentum	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$
L	Self-inductance	H
m	Mass	kg
M	Magnification	(unitless)
M	Mutual inductance	H
MA	Mechanical Advantage	(unitless)
ME	Mechanical Energy	J
n	Index of refraction	(unitless)
p	Object distance	m
$\vec{p}$	Momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
P	Power	W
PE or U	Potential Energy	J
$PE_{elastic}$	Elastic potential energy	J
$PE_{electric}$	Electrical potential energy	J
PE_g	Gravitational potential energy	J
q	Image distance	m
q or Q	Charge	C
Q	Heat, Entropy	J
R	Radius of curvature	m
R	Resistance	Ω
s	Arc length	m
t	Time	s
Δt	Time interval	s
$\vec{v}$	Velocity	$\frac{\text{m}}{\text{s}}$
v_t	Tangential speed	$\frac{\text{m}}{\text{s}}$
$\vec{v}_x$	Velocity in the x direction	$\frac{\text{m}}{\text{s}}$
$\vec{v}_y$	Velocity in the y direction	$\frac{\text{m}}{\text{s}}$
V	Electric potential	V
ΔV	Electric potential difference	V
V	Volume	m^3
W	Work	J
x or y	Position	m

Constants

Symbol	Name	Established Value	Value Used
ϵ_0	Permittivity of a vacuum	$8.854\ 187\ 817 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$	$8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$
ϕ	Golden ratio	1.618 033 988 749 894 848 20	
π	Archimedes' constant	3.141 592 653 589 793 238 46	
g, a_g	Gravitational acceleration constant	$9.79171\ \frac{m}{s^2}$ (varies by location)	$9.81\ \frac{m}{s^2}$
c	Speed of light in a vacuum	$2.997\ 924\ 58 \times 10^8\ \frac{m}{s}$ (exact)	$3.00 \times 10^8\ \frac{m}{s}$
e	Natural logarithmic base	2.718 281 828 459 045 235 36	
e^-	Elementary charge	$1.602\ 177\ 33 \times 10^{19}$ C	1.60×10^{19} C
G	Gravitational constant	$6.672\ 59 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$	$6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$
k_C	Coulomb's constant	$8.987\ 551\ 788 \times 10^9\ \frac{N \cdot m^2}{C^2}$	$8.99 \times 10^9\ \frac{N \cdot m^2}{C^2}$
N_A	Avogadro's constant	$6.022\ 141\ 5 \times 10^{23}\ mol^{-1}$	

Astronomical Data

Symbol	Object	Mean Radius	Mass	Mean Orbit Radius	Orbital Period
☽	Moon	1.74×10^6 m	7.36×10^{22} kg	3.84×10^8 m	2.36×10^6 s
☉	Sun	6.96×10^8 m	1.99×10^{30} kg	—	—
☿	Mercury	2.43×10^6 m	3.18×10^{23} kg	5.79×10^{10} m	7.60×10^6 s
♀	Venus	6.06×10^6 m	4.88×10^{24} kg	1.08×10^{11} m	1.94×10^7 s
♁	Earth	6.37×10^6 m	5.98×10^{24} kg	1.496×10^{11} m	3.156×10^7 s
♂	Mars	3.37×10^6 m	6.42×10^{23} kg	2.28×10^{11} m	5.94×10^7 s
	Ceres[1]	4.71×10^5 m	9.5×10^{20} kg	4.14×10^{11} m	1.45×10^8 s
♃	Jupiter	6.99×10^7 m	1.90×10^{27} kg	7.78×10^{11} m	3.74×10^8 s
♄	Saturn	5.85×10^7 m	5.68×10^{26} kg	1.43×10^{12} m	9.35×10^8 s
♅	Uranus	2.33×10^7 m	8.68×10^{25} kg	2.87×10^{12} m	2.64×10^9 s
♆	Neptune	2.21×10^7 m	1.03×10^{26} kg	4.50×10^{12} m	5.22×10^9 s
♇	Pluto[1]	1.15×10^6 m	1.31×10^{22} kg	5.91×10^{12} m	7.82×10^9 s
	Eris[2,1]	2.4×10^6 m	1.5×10^{22} kg	1.01×10^{13} m	1.75×10^{10} s

[1] Ceres, Pluto, and Eris are classified as "Dwarf Planets" by the IAU
[2] Eris was formerly known as 2003 UB_{313}

Mathematics Review for Physics

This is a summary of the most important parts of mathematics as we will use them in a physics class. There are numerous parts that are completely omitted, others are greatly abridged. Do not assume that this is a complete coverage of any of these topics.

Algebra

Fundamental properties of algebra

$a + b = b + a$	Commutative law for addition
$(a + b) + c = a + (b + c)$	Associative law for addition
$a + 0 = 0 + a = a$	Identity law for addition
$a + (-a) = (-a) + a = 0$	Inverse law for addition
$ab = ba$	Commutative law for multiplication
$(ab)c = a(bc)$	Associative law for multiplication
$(a)(1) = (1)(a) = a$	Identity law for multiplication
$a\frac{1}{a} = \frac{1}{a}a = 1$	Inverse law for multiplication
$a(b + c) = ab + ac$	Distributive law

Exponents

$(ab)^n = a^n b^n$ $\quad$ $(a/b)^n = a^n/b^n$

$a^n a^m = a^{n+m}$ $\quad$ $0^n = 0$

$a^n/a^m = a^{n-m}$ $\quad$ $a^0 = 1$

$(a^n)^m = a^{(mn)}$ $\quad$ $0^0 = 1$ (by definition)

Logarithms

$$x = a^y \rightarrow y = \log_a x$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^n) = n \log_a x$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a} = (\log_b x)(\log_a b)$$

Binomial Expansions

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

$$(a + b)^n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a^i b^{n-i}$$

Quadratic formula

For equations of the form $ax^2 + bx + c = 0$ the solutions are:

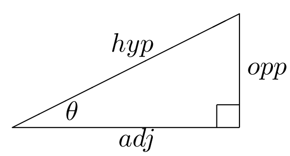
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Geometry

Shape	Area	Volume
Triangle	$A = \frac{1}{2}bh$	—
Rectangle	$A = lw$	—
Circle	$A = \pi r^2$	—
Rectangular prism	$A = 2(lw + lh + hw)$	$V = lwh$
Sphere	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Cylinder	$A = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Cone	$A = \pi r\sqrt{r^2 + h^2} + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$

Trigonometry

In physics only a small subset of what is covered in a trigonometry class is likely to be used, in particular sine, cosine, and tangent are useful, as are their inverse functions. As a reminder, the relationships between those functions and the sides of a right triangle are summarized as follows:



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

The inverse functions are only defined over a limited range. The $\tan^{-1} x$ function will yield a value in the range $-90° < \theta < 90°$, $\sin^{-1} x$ will be in $-90° \le \theta \le 90°$, and $\cos^{-1} x$ will yield one in $0° \le \theta \le 180°$. Care must be taken to ensure that the result given by a calculator is in the correct quadrant, if it is not then an appropriate correction must be made.

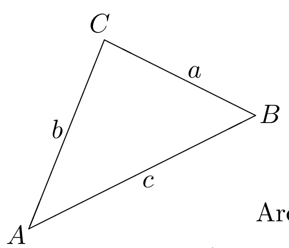
Degrees	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

$$\sin^2\theta + \cos^2\theta = 1 \qquad 2\sin\theta\cos\theta = \sin(2\theta)$$

Trigonometric functions in terms of each other

$\sin\theta =$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$
$\cos\theta =$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$
$\tan\theta =$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$
$\sec\theta =$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$
$\cot\theta =$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$
$\sin\theta =$	$\frac{1}{\csc\theta}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\frac{1}{\sqrt{\csc^2\theta-1}}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\csc\theta$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\sqrt{\csc^2\theta-1}$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Law of sines, law of cosines, area of a triangle



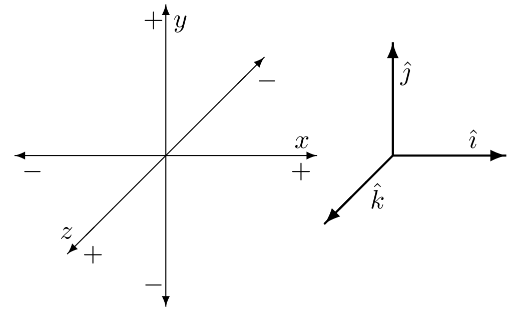
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\text{let } s = \tfrac{1}{2}(a+b+c)$$
$$\text{Area}= \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area}= \tfrac{1}{2}bc\sin A = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}ab\sin C$$

Vectors

A **vector** is a quantity with both magnitude and direction, such as displacement or velocity. Your textbook indicates a vector in bold-face type as **V** and in class we have been using $\vec{V}$. Both notations are equivalent.

A **scalar** is a quantity with only magnitude. This can either be a quantity that is directionless such as time or mass, or it can be the magnitude of a vector quantity such as speed or distance traveled. Your textbook indicates a scalar in italic type as V, in class we have not done anything to distinguish a scalar quantity. The magnitude of $\vec{V}$ is written as V or $|\vec{V}|$.

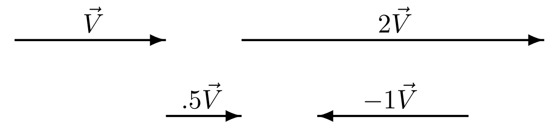
A **unit vector** is a vector with magnitude 1 (a dimensionless constant) pointing in some significant direction. A unit vector pointing in the direction of the vector $\vec{V}$ is indicated as $\hat{V}$ and would commonly be called V-hat. Any vector can be normalized into a unit vector by dividing it by its magnitude, giving $\hat{V} = \frac{\vec{V}}{V}$. Three special unit vectors, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are introduced with chapter 3. They point in the directions of the positive x, y, and z axes, respectively (as shown below).



Vectors can be added to other vectors of the same dimension (i.e. a velocity vector can be added to another velocity vector, but not to a force vector). The sum of all vectors to be added is called the **resultant** and is equivalent to all of the vectors combined.

Multiplying Vectors

Any vector can be multiplied by any scalar, this has the effect of changing the magnitude of the vector but not its direction (with the exception that multiplying a vector by a negative scalar will reverse the direction of the vector). As an example, multiplying a vector $\vec{V}$ by several scalars would give:



In addition to scalar multiplication there are also two ways to multiply vectors by other vectors. They will not be directly used in class but being familiar with them may help to understand how some physics equations are derived. The first, the **dot product** of vectors $\vec{V}_1$ and $\vec{V}_2$, represented as $\vec{V}_1 \cdot \vec{V}_2$ measures the tendency of the two vectors to point in the same direction. If the angle between the two vectors is θ the dot product yields a scalar value as

$$\vec{V}_1 \cdot \vec{V}_2 = V_1 V_2 \cos\theta$$

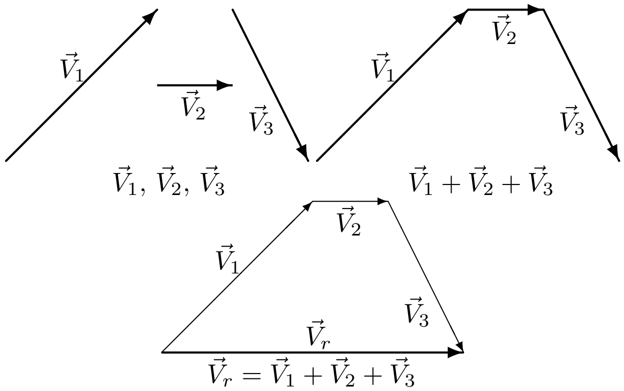
The second method of multiplying two vectors, the **cross product**, (represented as $\vec{V}_1 \times \vec{V}_2$) measures the tendency of vectors to be perpendicular to each other. It yields a third vector perpendicular to the two original vectors with magnitude

$$|\vec{V}_1 \times \vec{V}_2| = V_1 V_2 \sin\theta$$

The direction of the cross product is perpendicular to the two vectors being crossed and is found with the right-hand rule — point the fingers of your right hand in the direction of the first vector, curl them toward the second vector, and the cross product will be in the direction of your thumb.

Adding Vectors Graphically

The sum of any number of vectors can be found by drawing them head-to-tail to scale and in proper orientation then drawing the resultant vector from the tail of the first vector to the point of the last one. If the vectors were drawn accurately then the magnitude and direction of the resultant can be measured with a ruler and protractor. In the example below the vectors $\vec{V}_1$, $\vec{V}_2$, and $\vec{V}_3$ are added to yield $\vec{V}_r$

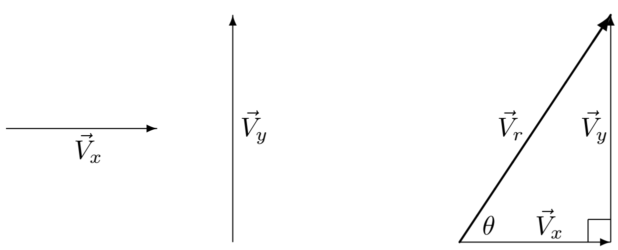


Adding Parallel Vectors

Any number of parallel vectors can be directly added by adding their magnitudes if one direction is chosen as positive and vectors in the opposite direction are assigned a negative magnitude for the purposes of adding them. The sum of the magnitudes will be the magnitude of the resultant vector in the positive direction, if the sum is negative then the resultant will point in the negative direction.

Adding Perpendicular Vectors

Perpendicular vectors can be added by drawing them as a right triangle and then finding the magnitude and direction of the hypotenuse (the resultant) through trigonometry and the Pythagorean theorem. If $\vec{V}_r = \vec{V}_x + \vec{V}_y$ and $\vec{V}_x \perp \vec{V}_y$ then it works as follows:



Since the two vectors to be added and the resultant form a right triangle with the resultant as the hypotenuse the Pythagorean theorem applies giving

$$V_r = |\vec{V}_r| = \sqrt{V_x^2 + V_y^2}$$

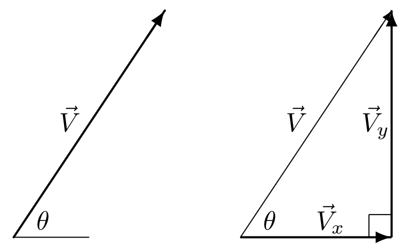
The angle θ can be found by taking the inverse tangent of the ratio between the magnitudes of the vertical and horizontal vectors, thus

$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

As was mentioned above, care must be taken to ensure that the angle given by the calculator is in the appropriate quadrant for the problem, this can be checked by looking at the diagram drawn to solve the problem and verifying that the answer points in the direction expected, if not then make an appropriate correction.

Resolving a Vector Into Components

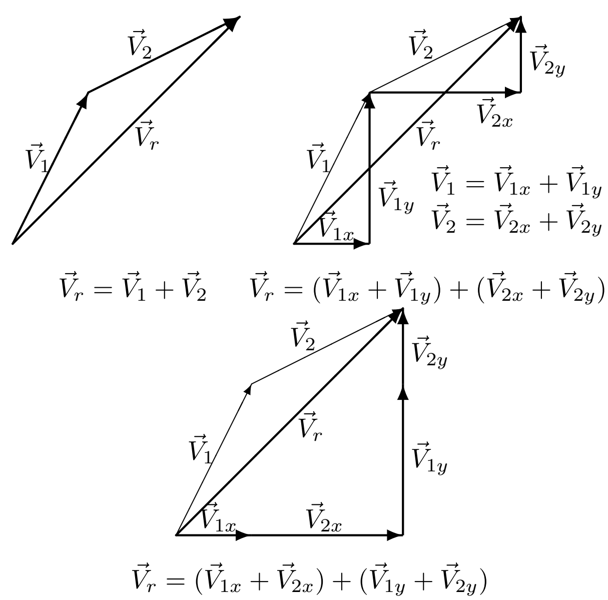
Just two perpendicular vectors can be added to find a single resultant, any single vector $\vec{V}$ can be resolved into two perpendicular **component vectors** $\vec{V}_x$ and $\vec{V}_y$ so that $\vec{V} = \vec{V}_x + \vec{V}_y$.



As the vector and its components can be drawn as a right triangle the ratios of the sides can be found with trigonometry. Since $\sin\theta = \frac{V_y}{V}$ and $\cos\theta = \frac{V_x}{V}$ it follows that $V_x = V\cos\theta$ and $V_y = V\sin\theta$ or in a vector form, $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$. (This is actually an application of the dot product, $\vec{V}_x = (\vec{V}\cdot\hat{\imath})\hat{\imath}$ and $\vec{V}_y = (\vec{V}\cdot\hat{\jmath})\hat{\jmath}$, but it is not necessary to know that for this class)

Adding Any Two Vectors Algebraically

Only vectors with the same direction can be directly added, so if vectors pointing in multiple directions must be added they must first be broken down into their components, then the components are added and resolved into a single resultant vector — if in two dimensions $\vec{V}_r = \vec{V}_1 + \vec{V}_2$ then



Once the sums of the component vectors in each direction have been found the resultant can be found from them just as an other perpendicular vectors may be added. Since from the last figure $\vec{V}_r = (\vec{V}_{1x}+\vec{V}_{2x})+(\vec{V}_{1y}+\vec{V}_{2y})$ and it was previously established that $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$ it follows that

$$\vec{V}_r = (V_1\cos\theta_1 + V_2\cos\theta_2)\,\hat{\imath} + (V_1\sin\theta_1 + V_2\sin\theta_2)\,\hat{\jmath}$$

and

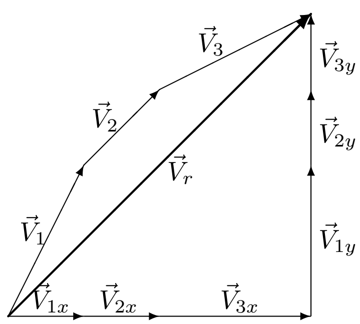
$$\begin{aligned} V_r &= \sqrt{(V_{1x}+V_{2x})^2 + (V_{1y}+V_{2y})^2} \\ &= \sqrt{(V_1\cos\theta_1 + V_2\cos\theta_2)^2 + (V_1\sin\theta_1 + V_2\sin\theta_2)^2} \end{aligned}$$

with the direction of the resultant vector $\vec{V}_r$, θ_r, being found with

$$\theta_r = \tan^{-1}\frac{V_{1y}+V_{2y}}{V_{1x}+V_{2x}} = \tan^{-1}\frac{V_1\sin\theta_1 + V_2\sin\theta_2}{V_1\cos\theta_1 + V_2\cos\theta_2}$$

Adding Any Number of Vectors Algebraically

For a total of n vectors $\vec{V}_i$ being added with magnitudes V_i and directions θ_i the magnitude and direction are:



$$\vec{V}_r = \left(\sum_{i=1}^{n} V_i\cos\theta_i\right)\hat{\imath} + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)\hat{\jmath}$$

$$V_r = \sqrt{\left(\sum_{i=1}^{n} V_i\cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)^2}$$

$$\theta_r = \tan^{-1}\frac{\sum_{i=1}^{n} V_i\sin\theta_i}{\sum_{i=1}^{n} V_i\cos\theta_i}$$

(The figure shows only three vectors but this method will work with any number of them so long as proper care is taken to ensure that all angles are measured the same way and that the resultant direction is in the proper quadrant.)

Calculus

Although this course is based on algebra and not calculus, it is sometimes useful to know some of the properties of derivatives and integrals. If you have not yet learned calculus it is safe to skip this section.

Derivatives

The **derivative** of a function $f(t)$ with respect to t is a function equal to the slope of a graph of $f(t)$ vs. t at every point, assuming that slope exists. There are several ways to indicate that derivative, including:

$$\frac{\mathrm{d}}{\mathrm{d}t}f(t)$$

$$f'(t)$$

$$\dot{f}(t)$$

The first one from that list is unambiguous as to the independent variable, the others assume that there is only one variable, or in the case of the third one that the derivative is taken with respect to time. Some common derivatives are:

$$\frac{\mathrm{d}}{\mathrm{d}t}t^n = nt^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin t = \cos t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\cos t = -\sin t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}e^t = e^t$$

If the function is a compound function then there are a few useful rules to find its derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}cf(t) = c\frac{\mathrm{d}}{\mathrm{d}t}f(t) \qquad c \text{ constant for all } t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[f(t)+g(t)] = \frac{\mathrm{d}}{\mathrm{d}t}f(t) + \frac{\mathrm{d}}{\mathrm{d}t}g(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}f(u) = \frac{\mathrm{d}}{\mathrm{d}t}u\frac{\mathrm{d}}{\mathrm{d}u}f(u)$$

Integrals

The **integral**, or **antiderivative** of a function $f(t)$ with respect to t is a function equal to the the area under a graph of $f(t)$ vs. t at every point plus a constant, assuming that $f(t)$ is continuous. Integrals can be either indefinite or definite. An indefinite integral is indicated as:

$$\int f(t)\,\mathrm{d}t$$

while a definite integral would be indicated as

$$\int_a^b f(t)\,\mathrm{d}t$$

to show that it is evaluated from $t = a$ to $t = b$, as in:

$$\int_a^b f(t)\,\mathrm{d}t = \int f(t)\,\mathrm{d}t\Big|_{t=a}^{t=b} = \int f(t)\,\mathrm{d}t\Big|_{t=b} - \int f(t)\,\mathrm{d}t\Big|_{t=a}$$

Some common integrals are:

$$\int t^n\,\mathrm{d}t = \frac{t^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int t^{-1}\,\mathrm{d}t = \ln t + C$$
$$\int \sin t\,\mathrm{d}t = -\cos t + C$$
$$\int \cos t\,\mathrm{d}t = \sin t + C$$
$$\int e^t\,\mathrm{d}t = e^t + C$$

There are rules to reduce integrals of some compound functions to simpler forms (there is no general rule to reduce the integral of the product of two functions):

$$\int cf(t)\,\mathrm{d}t = c\int f(t)\,\mathrm{d}t \qquad c \text{ constant for all } t$$
$$\int [f(t) + g(t)]\,\mathrm{d}t = \int f(t)\,\mathrm{d}t + \int g(t)\,\mathrm{d}t$$

Series Expansions

Some functions are difficult to work with in their normal forms, but once converted to their series expansion can be manipulated easily. Some common series expansions are:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i+1}}{(2i+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i}}{(2i)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{i=0}^{\infty}\frac{x^i}{(i)!}$$

Physics Using Calculus

In our treatment of mechanics the majority of the equations that have been covered in the class have been either approximations or special cases where the acceleration or force applied have been held constant. This is required because the class is based on algebra and to do otherwise would require the use of calculus.

None of what is presented here is a required part of the class, it is here to show how to handle cases outside the usual approximations and simplifications. Feel free to skip this section, none of it will appear as a required part of any assignment or test in this class.

Notation

Because the letter 'd' is used as a part of the notation of calculus the variable 'r' is often used to represent the position vector of the object being studied. (Other authors use 's' for the spatial position or generalize 'x' to two or more dimensions.) In a vector form that would become $\vec{r}$ to give both the magnitude and direction of the position. It is assumed that the position, velocity, acceleration, etc. can be expressed as a function of time as $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ but the dependency on time is usually implied and not shown explicitly.

Translational Motion

Many situations will require an acceleration that is not constant. Everything learned about translational motion so far used the simplification that the acceleration remained constant, with calculus we can let the acceleration be any function of time.

Since velocity is the slope of a graph of position vs. time at any point, and acceleration is the slope of velocity vs. time these can be expressed mathematically as derivatives:

$$\vec{v} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}$$

$$\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}$$

The reverse of this relationship is that the displacement of an object is equal to the area under a velocity vs. time graph and velocity is equal to the area under an acceleration vs. time graph. Mathematically this can be expressed as integrals:

$$\vec{v} = \int \vec{a}\,\mathrm{d}t$$

$$\vec{r} = \int \vec{v}\,\mathrm{d}t = \iint \vec{a}\,\mathrm{d}t^2$$

Using these relationships we can derive the main equations of motion that were introduced by starting with the integral of a constant velocity $\vec{a}(t) = \vec{a}$ as

$$\int_{t_i}^{t_f} \vec{a}\,\mathrm{d}t = \vec{a}\Delta t + C$$

The constant of integration, C, can be shown to be the initial velocity, $\vec{v}_i$, so the entire expression for the final velocity becomes

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

or

$$\vec{v}_f(t) = \vec{v}_i + \vec{a}t$$

Similarly, integrating that expression with respect to time gives an expression for position:

$$\int_{t_i}^{t_f} (\vec{v}_i + \vec{a}t)\,\mathrm{d}t = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2 + C$$

Once again the constant of integration, C, can be shown to be the initial position, $\vec{r}_i$, yielding an expression for position vs. time

$$\vec{r}_f(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

While calculus can be used to derive the algebraic forms of the equations the reverse is not true. Algebra can handle a subset of physics and only at the expense of needing to remember separate equations for various special cases. With calculus the few definitions shown above will suffice to predict any linear motion.

Work and Power

The work done on an object is the integral of dot product of the force applied with the path the object takes, integrated as a contour integral over the path.

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The power developed is the time rate of change of the work done, or the derivative of the work with respect to time.

$$P = \frac{\mathrm{d}}{\mathrm{d}t}W$$

Momentum

Newton's second law of motion changes from the familiar $\vec{F} = m\vec{a}$ to the statement that the force applied to an object is the time derivative of the object's momentum.

$$\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{p}$$

Similarly, the impulse delivered to an object is the integral of the force applied in the direction parallel to the motion with respect to time.

$$\Delta\vec{p} = \int \vec{F}\,\mathrm{d}t$$

Rotational Motion

The equations for rotational motion are very similar to those for translational motion with appropriate variable substitutions. First, the angular velocity is the time derivative of the angular position.

$$\vec{\omega} = \frac{\mathrm{d}\vec{\theta}}{\mathrm{d}t}$$

The angular acceleration is the time derivative of the angular velocity or the second time derivative of the angular position.

$$\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\theta}}{\mathrm{d}t^2}$$

The angular velocity is also the integral of the angular acceleration with respect to time.

$$\vec{\omega} = \int \vec{\alpha}\,\mathrm{d}t$$

The angular position is the integral of the angular velocity with respect to time or the second integral of the angular acceleration with respect to time.

$$\vec{\theta} = \int \vec{\omega}\,\mathrm{d}t = \iint \vec{\alpha}\,\mathrm{d}t^2$$

The torque exerted on an object is the cross product of the radius vector from the axis of rotation to the point of action with the force applied.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The moment of inertia of any object is the integral of the square of the distance from the axis of rotation to each element of the mass over the entire mass of the object.

$$I = \int r^2\,\mathrm{d}m$$

The torque applied to an object is the time derivative of the object's angular momentum.

$$\vec{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

The integral of the torque with respect to time is the angular momentum of the object.

$$\vec{L} = \int \vec{\tau}\,\mathrm{d}t$$

Data Analysis

Comparative Measures

Percent Error

When experiments are conducted where there is a calculated result (or other known value) against which the observed values will be compared the percent error between the observed and calculated values can be found with

$$\textit{percent error} = \left| \frac{\textit{observed value} - \textit{accepted value}}{\textit{accepted value}} \right| \times 100\%$$

Percent Difference

When experiments involve a comparison between two experimentally determined values where neither is regarded as correct then rather than the percent error the percent difference can be calculated. Instead of dividing by the accepted value the difference is divided by the mean of the values being compared. For any two values, a and b, the percent difference is

$$\textit{percent difference} = \left| \frac{a-b}{\frac{1}{2}(a+b)} \right| \times 100\%$$

Dimensional Analysis

Unit Conversions

When converting units you need to be careful to ensure that conversions are done properly and not accidentally reversed (or worse). The easiest way to do this is to be careful to always keep the units associated with each number and at each step always multiply by something equal to 1.

As an example, if you have a distance of 78.25 km and want to convert it to meters the immediate reaction is to say that you can multiply by 1000, or is that divide by 1000. Knowing that 1 km = 1000 m it is obvious then that $\frac{1\text{ km}}{1000\text{ m}} = 1 = \frac{1000\text{ m}}{1\text{ km}}$. Since both of them are equal to 1 they can be multiplied by what you wish to convert without changing the quantity itself, it will only change the units in which it is shown. So, to convert 78.25 km to meters you would multiply by $\frac{1000\text{ m}}{1\text{ km}}$:

$$(78.25\ \not{\text{km}})\left(\frac{1000\text{ m}}{1\ \not{\text{km}}}\right) = 78\ 250\text{ m}$$

As you can see, the km in the original quantity and in the denominator cancel leaving meters. If you were to reverse the conversion accidentally you would get this:

$$(78.25\text{ km})\left(\frac{1\text{ km}}{1000\text{ m}}\right) = 0.07825\ \frac{\text{km}^2}{\text{m}}$$

Instead of canceling the km and leaving the measurement in meters the result is in $\frac{\text{km}^2}{\text{m}}$, a meaningless quantity. The nonsensical result is an indication that the conversion was reversed and should be redone. If the conversion was done without the units then it could be reversed without realizing that it happened.

Units on Constants

Consider a mass vibrating on the end of a spring. The equation relating the mass to the frequency on graph might look something like

$$y = \frac{3.658}{x^2} - 1.62$$

This isn't what we're looking for, a first pass would be to change the x and y variables into f and m for frequency and mass, this would then give

$$m = \frac{3.658}{f^2} - 1.62$$

This is progress but there's still a long way to go. Mass is measured in kg and frequency is measured in s^{-1}, since these variables represent the entire measurement (including the units) and not just the numeric portion they do not need to be labeled, but the constants in the equation do need to have the correct units applied.

To find the units it helps to first re-write the equation using just the units, letting some variable such as u (or u_1, u_2, u_3, etc.) stand for the unknown units. The example equation would then become

$$\text{kg} = \frac{u_1}{(\text{s}^{-1})^2} + u_2$$

Since any quantities being added or subtracted must have the same units the equation can be split into two equations

$$\text{kg} = \frac{u_1}{(\text{s}^{-1})^2} \quad \text{and} \quad \text{kg} = u_2$$

The next step is to solve for u_1 and u_2, in this case $u_2 = \text{kg}$ is immediately obvious, u_1 will take a bit more effort. A first simplification yields

$$\text{kg} = \frac{u_1}{\text{s}^{-2}}$$

then multiplying both sides by s^{-2} gives

$$(\text{s}^{-2})(\text{kg}) = \left(\frac{u_1}{\text{s}^{-2}}\right)(\text{s}^{-2})$$

which finally simplifies and solves to

$$u_1 = \text{kg} \cdot \text{s}^{-2}$$

Inserting the units into the original equation finally yields

$$m = \frac{3.658\text{ kg} \cdot \text{s}^{-2}}{f^2} - 1.62\text{ kg}$$

which is the final equation with all of the units in place. Checking by choosing a frequency and carrying out the calculations in the equation will show that it is dimensionally consistent and yields a result in the units for mass (kg) as expected.

Final Exam Description

There will be 100 questions on the final exam. None of them will directly be from chapters 1–11 but topics covered in those chapters form a foundation for later topics.

Category	Topics	Chapters	# of Questions	% of Test	Total
Circular Motion	Gravitation & Satellites	12, 13, 14	9	9%	9%
Waves	Wave Mechanics	25	11	11%	22%
	Sound	26	11	11%	
Optics	Light & Color	27, 28	11	11%	38%
	Reflection & Mirrors	29	5	5%	
	Refraction & Lenses	29, 30	13	13%	
	Diffraction & Interference	31	9	9%	
Electricity	Electrostatics	32	5	5%	20%
	Electric Fields & Potential	33	5	5%	
	Current	34	5	5%	
	Circuits	35	5	5%	
Magnetism	Magnetism	36	6	6%	11%
	Induction	37	5	5%	

Final Exam Equation Sheet

This is a slightly reduced copy of the equation sheet that you will receive with the final exam.

Physics equations

*Use **9.8 m/s²** or **10 m/s/s** for the acceleration due to gravity on earth

Horizontal:

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$a_x = \frac{\Delta v_x}{\Delta t}$$

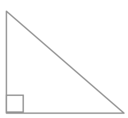
Vertical:

$$\Delta v_y = g\Delta t$$

$$d_y = \frac{1}{2} g\Delta t^2$$

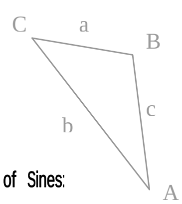
$$c^2 = a^2 + b^2$$

SOHCAHTOA



Law of Cosines:

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$



Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$w = mg$$

$$\Sigma F = ma$$

$$P_{ressure} = \frac{F}{A}$$

$$\Delta p = Ft = \Delta(mv)$$

$$(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f$$

$$(m_1v_1)_i + (m_2v_2)_i = (mv)_f$$

$$(mv)_i = (m_1v_1)_f + (m_2v_2)_f$$

$$W = Fd\cos\theta = \Delta KE$$

$$P = \frac{W}{\Delta t}$$

$$AMA = \frac{F_{out}}{F_{in}}$$

$$IMA = \frac{d_{in}}{d_{out}}$$

$$Efficiency = \frac{W_{out}}{W_{in}} \times 100 = \frac{AMA}{IMA}$$

$$PE = mgh$$

$$KE = \tfrac{1}{2}mv^2$$

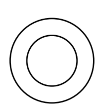
$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r} = ma_c$$

$$\tau = F_\perp \times leverarm$$

$$I = mr^2$$



$$L = I \times \omega = mvr$$

$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$v = f\lambda$$

$$f' = f_o\left(\frac{v \pm v_o}{v \pm v_s}\right)$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_1}{n_2}$$

$$F = k\frac{q_1q_2}{d^2}$$

$$I = \frac{q}{t}$$

$$V = \frac{PE}{q}$$

$$V = IR$$

$$C = \frac{q}{V}$$

$$P = \frac{W}{t} = \frac{qV}{t} = IV$$

Note that some equations may be slightly different than the ones we have used in class, the sheet that you receive will have these forms of the equations, it will be up to you to know how to use them or to know other forms that you are able to use.