Honors Physics Review Notes

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The most recent version of this can be found at http://www.tomstrong.org/physics/

These notes are meant to be a summary of important points covered in the Honors Physics class at Mt. Lebanon High School. They are not meant to be a replacement for your own notes that you take in class, nor are they a replacement for your textbook. Much of the material in here is taken from the textbook without specifically acknowledging each case, in particular the organization and overall structure exactly match the 2002 edition of *Holt Physics* by Serway and Faughn and many of the expressions of the ideas come from there as well.

The mixed review exercises were taken from the supplementary materials provided with the textbook. They are a representative sampling of the type of mathematical problems you may see on the final exam for the course. There will also be conceptual questions on the exam that may not be covered by the exercises included here. These exercises are provided to help you to review material that has not been seen in some time, they are not meant to be your only resource for studying. Exercises are included from chapters 1–8, 12–17, and 19–22.

The answers at the end of the review are taken from the textbook, often without verifying that they are correct. Use them to help you to solve the problems but do not accept them as correct without verifying them yourself.

This is a work in progress and will be changing and expanding over time. I have attempted to verify the correctness of the information presented here, but the final responsibility there is yours. Before relying on the information in these notes please verify it against other sources.

Chapter 1 — The Science of Physics

1.1 What is Physics?

Some major areas of Physics:

Mechanics — motion and its causes — falling objects, friction, weight

Thermodynamics — heat and temperature — melting and freezing processes, engines, refrigerators

Vibrations and Waves — specific types of repeating motions — springs, pendulums, sound

Optics — light — mirrors, lenses, color

Electromagnetism — electricity, magnetism, and light — electrical charge, circuitry, magnets

Relativity — particles moving at very high speeds — particle accelerators, particle collisions, nuclear energy

Quantum Mechanics — behavior of sub-microscopic particles — the atom and its parts

The steps of the Scientific Method

1. Make observations and collect data that lead to a question
2. Formulate and objectively test hypotheses by experiments (sometimes listed as 2 steps)
3. Interpret results and revise the hypotheses if necessary
4. State conclusions in a form that can be evaluated by others

1.2 Measurements in Experiments

Measurements

There are 7 basic dimensions in SI (Système International), the 3 we will use most often are:

- Length — **meter** (m) — was 1/10,000,000 of the distance from the equator to the North Pole — now the distance traveled by light in 3.3×10^{-9} s
- Mass — **kilogram** (kg) — was the mass of 0.001 cubic meters of water, now the mass of a specific platinum-iridium cylinder
- Time — **second** (s) — was a fraction of a mean solar day, now 9,162,631,700 times the period of a radio wave emitted by a Cesium-133 atom

Common SI Prefixes

Prefix	Multiple	Abbrev.			
nano-	10^{-9}	n	deci-	10^{-1}	d
micro-	10^{-6}	μ	kilo-	10^{3}	k
milli-	10^{-3}	m	mega-	10^{6}	M
centi-	10^{-2}	c	giga-	10^{9}	G

Accuracy vs. Precision

- **Accuracy** describes how close a measured value is to the true value of the quantity being measured

  Problems with accuracy are due to error. To avoid error:

  - Take repeated measurements to be certain that they are consistent (avoid human error)
  - Take each measurement in the same way (avoid method error)
  - Be sure to use measuring equipment in good working order (avoid instrument error)

- **Precision** refers to the degree of exactness with which a measurement is made and stated.

  - 1.325 m is more precise than 1.3 m
  - lack of precision is usually a result of the limitations of the measuring instrument, not human error or lack of calibration
  - You can estimate where divisions would fall between the marked divisions to increase the precision of the measurement

1.3 The Language of Physics

There are many symbols that will be used in this class, some of the more common will be:

Symbol	Meaning
Δx	Change in x
x_i, x_f	Initial, final values of x
$\sum F$	Sum of all F

Dimensional analysis provides a way of checking to see if an equation has been set up correctly. If the units resulting from the calculation are not those that are expected then it's very unlikely that the numbers will be correct either. **Order of magnitude estimates** provide a quick way to evaluate the appropriateness of an answer — if the estimate doesn't match the answer then there's an error somewhere.

Counting Significant Figures in a Number

Rule	Example
All counted numbers have an infinite number of significant figures	10 items, 3 measurements
All mathematical constants have an infinite number of significant figures	$1/2$, π, e
All nonzero digits are significant	42 has two significant figures; 5.236 has four
Always count zeros between nonzero digits	20.08 has four significant figures; 0.00100409 has six
Never count leading zeros	042 and 0.042 both have two significant figures
Only count trailing zeros if the number contains a decimal point	4200 and 420000 both have two significant figures; 420. has three; 420.00 has five
For numbers in scientific notation apply the above rules to the mantissa (ignore the exponent)	4.2010×10^{28} has five significant figures

Counting Significant Figures in a Calculation

Rule	Example
When adding or subtracting numbers, find the number which is known to the fewest decimal places, then round the result to that decimal place.	$21.398 + 405 - 2.9 = 423$ (3 significant figures, rounded to the ones position)
When multiplying or dividing numbers, find the number with the fewest significant figures, then round the result to that many significant figures.	$0.049623 \times 32.0/478.8 = 0.00332$ (3 significant figures)
When raising a number to some power count the number's significant figures, then round the result to that many significant figures.	$5.8^2 = 34$ (2 significant figures)
Mathematical constants do not influence the precision of any computation.	$2 \times \pi \times 4.00 = 25.1$ (3 significant figures)
In order to avoid introducing errors during multi-step calculations, keep extra significant figures for intermediate results then round properly when you reach the final result.	

Rules for Rounding

Rule	Example
If the hundredths digit is 0 through 4 drop it and all following digits.	1.334 becomes 1.3
If the hundredths digit is 6 though 9 round the tenths digit up to the next higher value.	1.374 becomes 1.4
If the hundredths digit is a 5 followed by other non-zero digits then round the tenths digit up to the next higher value.	1.351 becomes 1.4
If the hundredths digit is a 5 not followed by any non-zero digits then if the tenths digit is even round down, if it is odd then round up.	1.350 becomes 1.4, 1.250 becomes 1.2
(assume that the result was to be rounded to the nearest 0.1, for other precisions adjust accordingly)	

Chapter 2 — Motion in One Dimension

2.1 Displacement and Velocity

The **displacement** of an object is the straight line (vector) drawn from the object's initial position to its new position. Displacement is independent of the path taken and is not necessarily the same as the distance traveled. Mathematically, displacement is:

$$\Delta x = x_f - x_i$$

The **average velocity**, equal to the constant velocity necessary to cover the given displacement in a certain time interval, is the displacement divided by the time interval during which the displacement occurred, measured in $\frac{\text{m}}{\text{s}}$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The **instantaneous velocity** of an object is equivalent to the slope of a tangent line to a graph of x vs. t at the time of interest.

The area under a graph of instantaneous velocity vs. time (v vs. t) is the displacement of the object during that time interval.

2.2 Acceleration

The **average acceleration** of an object is the rate of change of its velocity, measured in $\frac{\text{m}}{\text{s}^2}$. Mathematically, it is:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Like velocity, acceleration has both magnitude and direction. The speed of an object can increase or decrease with either positive or negative acceleration, depending on the direction of the velocity — negative acceleration does not always mean decrease in speed.

The **instantaneous acceleration** of an object is equivalent to the slope of a tangent line to the v vs. t graph at the time of interest, while the area under a graph of instantaneous acceleration vs. time (a vs. t) is the velocity of the object. In this class acceleration will almost always be constant in any problem.

Displacement and velocity with constant uniform acceleration can be expressed mathematically as any of:

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

2.3 Falling Objects

In the absence of air resistance all objects dropped near the surface of a planet fall with effectively the same constant acceleration — called **free fall**. That acceleration is always directed downward, so in the customary frame of reference it is negative, so:

$$a_g = g = -9.81\ \frac{\text{m}}{\text{s}^2}$$

Chapter 3 — Two-Dimensional Motion and Vectors

3.1 Introduction to Vectors

Vectors can be added graphically.

Vectors can be added in any order:

$$\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$$

To subtract a vector you add its opposite:

$$\vec{V}_1 - \vec{V}_2 = \vec{V}_1 + (-\vec{V}_2)$$

Multiplying a vector by a scalar results in a vector in same direction as the original vector with a magnitude equal to the original magnitude multiplied by the scalar.

3.2 Vector Operations

To add two perpendicular vectors use the Pythagorean Theorem to find the resultant magnitude and the inverse of the tangent function to find the direction: $V_r = \sqrt{V_x^2 + V_y^2}$ and the direction of V_r, θ_r, is $\tan^{-1} \frac{V_y}{V_x}$

Just as 2 perpendicular vectors can be added, any vector can be broken into two perpendicular component vectors: $\vec{V} = \vec{V}_x + \vec{V}_y$ where $\vec{V}_x = V \cos\theta \hat{\imath}$ and $\vec{V}_y = V \sin\theta \hat{\jmath}$;

Two vectors with the same direction can be added by adding their magnitudes, the resultant vector will have the same direction as the vectors that were added.

Any two (or more) vectors can be added by first decomposing them into component vectors, adding all of the x and y components together, and then adding the two remaining perpendicular vectors as described above.

3.3 Projectile Motion

The equations of motion introduced in chapter 2 are actually vector equations. Once the new symbol for displacement, $\vec{d} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$ has been introduced the most common ones can be rewritten as

$$\vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

Since motion in each direction is independent of motion in the other, objects moving in two dimensions are easier to analyze if each dimension is considered separately.

$$\vec{d}_x = \vec{v}_{xi}\Delta t + \frac{1}{2}\vec{a}_x \Delta t^2$$

$$\vec{d}_y = \vec{v}_{yi}\Delta t + \frac{1}{2}\vec{a}_y \Delta t^2$$

In the specific case of **projectile motion** there is no acceleration in the horizontal ($\hat{\imath}$) direction ($\vec{a}_x = 0$) and the acceleration in the vertical ($\hat{\jmath}$) direction is constant ($\vec{a}_y = \vec{a}_g = \vec{g} = -9.81 \frac{\mathrm{m}}{\mathrm{s}^2}\hat{\jmath}$). While $\vec{v}_y$ will change over time, $\vec{v}_x$ remains constant. Neglecting air resistance, the path followed by an object in projectile motion is a parabola. The following equations use these simplifications to describe the motion of a projectile launched with speed $\vec{v}_i$ and direction θ up from horizontal:

$$\vec{v}_x = v_i \cos\theta \hat{\imath} = \text{constant}$$

$$\vec{v}_{yi} = v_i \sin\theta \hat{\jmath}$$

$$\vec{d}_x = v_i \cos\theta \Delta t \hat{\imath}$$

$$\vec{d}_y = v_i \sin\theta \Delta t \hat{\jmath} + \frac{1}{2}\vec{g}\Delta t^2$$

To solve a projectile motion problem use one part of the problem to find Δt, then once you know Δt you can use that to solve the rest of the problem. In almost every case this will involve using the vertical part of the problem to find Δt which will then let you solve the horizontal part but there may be some problems where the opposite approach will be necessary. For the special case of a projectile launched on a level surface the range can be found with

$$d_x = \frac{v^2 \sin 2\theta}{a_g}$$

Chapter 4 — Forces and the Laws of Motion

4.1 Changes in Motion

Force causes change in velocity. It can cause a stationary object to move or a moving object to stop or otherwise change its motion.

The unit of force is the **newton** (N), equivalent to $\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$, which is defined as the amount of force that, when acting on a 1 kg mass, produces an acceleration of $1\frac{\text{m}}{\text{s}^2}$.

Contact forces act between any objects that are in physical contact with each other, while **field forces** act over a distance.

A **force diagram** is a diagram showing all of the forces acting on the objects in a system.

A **free-body diagram** is a diagram showing all of the forces acting on a single object isolated from its surroundings.

4.2 Newton's First Law

Newton's first law states that

> An object at rest remains at rest, and an object in motion continues in motion with constant velocity (that is, constant speed in a straight line) unless the object experiences a net external force.

This tendency of an object to not accelerate is called **inertia**. Another way of stating the first law is that if the net external force on an object is zero, then the acceleration of that object is also zero. Mathematically this is

$$\sum \vec{F} = 0 \longrightarrow \vec{a} = 0$$

An object experiencing no net external force is said to be in **equilibrium**, if it is also at rest then it is in **static equilibrium**

4.3 Newton's Second and Third Laws

Newton's second law states that

> The acceleration of an object is directly proportional to the net external force acting on the object and inversely proportional to the object's mass.

Mathematically this can be stated as:

$$\sum \vec{F} = m\vec{a}$$

Which in the case of no net external force ($\sum \vec{F} = 0$) also illustrates the first law:

$$\sum \vec{F} = 0 \longrightarrow m\vec{a} = 0 \longrightarrow \vec{a} = 0$$

Newton's third law states that

> If two objects interact, the magnitude of the force exerted on object 1 by object 2 is equal to the magnitude of the force simultaneously exerted on object 2 by object 1, and these two forces are opposite in direction.

Mathematically, this can be stated as:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

The two equal but opposite forces form an **action-reaction pair**.

4.4 Everyday Forces

The **weight** of an object ($\vec{F}_g$) is the gravitational force exerted on the object by the Earth. Mathematically:

$$\vec{F}_g = m\vec{g} \text{ where } \vec{g} = -9.81\frac{\text{m}}{\text{s}^2}\hat{j}$$

The **normal force** ($\vec{F}_n$) is the force exerted on an object by the surface upon which the object rests. This force is always perpendicular to the surface at the point of contact.

The force of **static friction** ($\vec{F}_s$) is the force that opposes motion before an object begins to move, it will prevent motion so long as it has a magnitude greater than the applied force in the direction of motion. The maximum magnitude of the force of static friction is the product of the magnitude of the normal force times the **coefficient of static friction** (μ_s) and its direction is opposite the direction of motion:

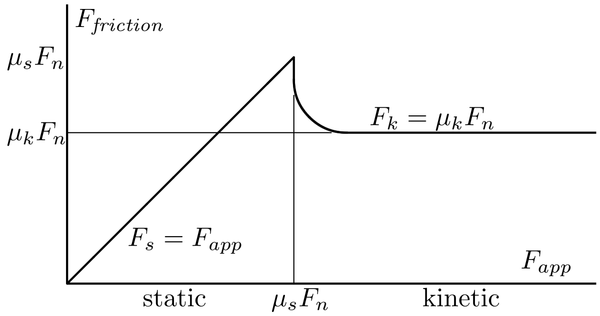
$$\mu_s = \frac{F_{s,max}}{F_n} \longrightarrow F_{s,max} = \mu_s F_n$$

The force of **kinetic friction** ($\vec{F}_k$) is the force that opposes the motion of an object that is sliding against a surface. The magnitude of the force of kinetic friction is the product of the magnitude of the normal force times the **coefficient of kinetic friction** (μ_k) and its direction is opposite the direction of motion:

$$\mu_k = \frac{F_k}{F_n} \longrightarrow F_k = \mu_k F_n$$

The force of friction between two solid objects depends only on the normal force and the coefficient of friction, it is independent of the the surface area in contact between them.

For an object at rest with a force applied to it the frictional force will vary as the applied force is increased.



Air resistance ($\vec{F}_R$) is the force that opposes the motion of an object though a fluid. For small speeds $F_R \propto v$ while for large speeds $F_R \propto v^2$. (Exactly what is small or large depends on things that are outside the scope of this class.) Air resistance is what will cause falling objects to eventually reach a **terminal speed** where $F_R = F_g$.

Solving Friction Problems

When solving friction problems start by drawing a diagram of the system being sure to include the gravitational force (F_g), normal force (F_n), frictional force (F_k, F_s, or $F_{s,max}$) and any applied force(s).

Determine an appropriate frame of reference, rotating the x and y coordinate axes if that is convenient for the problem (such as an object on an inclined plane). Resolve any vectors not lying along the coordinate axes into components.

Use Newton's second law ($\sum \vec{F} = m\vec{a}$) to find the relationship between the acceleration of the object (often zero) and the applied forces, setting up equations in both the x and y directions ($\sum \vec{F}_x = m\vec{a}_x$ and $\sum \vec{F}_y = m\vec{a}_y$) to solve for any unknown quantities.

Chapter 5 — Work and Energy

5.1 Work

A force that causes a displacement of an object does **work** (W) on the object. The work is equal to the product of the distance that the object is displaced times the component of the force in the direction of the displacement, or if θ is the angle between the displacement vector and the (constant) net applied force vector, then:

$$W = F_{net} d \cos\theta$$

The units of work are **Joules** (J) which are equivalent to $\mathrm{N \cdot m}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$. The sign of the work being done is significant, it is possible to do a negative amount of work.

5.2 Energy

Work done to change the speed of an object will accumulate as the **kinetic energy** (KE, or sometimes K) of the object. Kinetic energy is:

$$KE = \frac{1}{2}mv^2$$

If work is being done on an object, the **work-kinetic energy theorem** shows that:

$$W_{net} = \Delta KE$$

In addition to kinetic energy, there is also **potential energy** (PE, or sometimes U) which is the energy stored in an object because of its position. If the energy is stored by lifting the object to some height h, then the equation for **gravitational potential energy** is:

$$PE_g = mgh$$

Potential energy can also be stored in a compressed or stretched spring, if x is the distance that a spring is stretched from its rest position and k is the **spring constant** measuring the stiffness of the spring (in newtons per meter) then the **elastic potential energy** that is stored is:

$$PE_{elastic} = \frac{1}{2}kx^2$$

The units for all types of energy are the same as those for work, Joules.

5.3 Conservation of Energy

The sum an objects kinetic energy and potential energy is the object's **mechanical energy** (ME). In the absence of friction, the total mechanical energy of a system will remain the same. Mathematically, this can be expressed as:

$$ME_i = ME_f$$

In the case of a single object in motion, this becomes:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

5.4 Power

The rate at which energy is transferred is called **power** (P). The mathematical expression for power is:

$$P = \frac{W}{\Delta t} = \frac{Fd}{\Delta t} = F\frac{d}{\Delta t} = Fv$$

The units for power are **Watts** (W) which are equivalent to $\frac{\mathrm{J}}{\mathrm{s}}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}$

Chapter 6 — Momentum and Collisions

6.1 Momentum and Impulse

Momentum is a vector quantity described by the product of an object's mass times its velocity:

$$\vec{p} = m\vec{v}$$

If an object's momentum is known its kinetic energy can be found as follows:

$$KE = \frac{p^2}{2m}$$

The change in the momentum of an object is equal to the **impulse** delivered to the object. The impulse is equal to the constant net external force acting on the object times the time over which the force acts:

$$\Delta\vec{p} = \vec{F}\Delta t$$

Any force acting on on object will cause an impulse, including frictional and gravitational forces.

In one dimension the slope of a graph of the momentum of an object vs. time is the net external force acting on the object. The area under a graph of the net external force acting on an object vs. time is the total change in momentum of that object.

Momentum and impulse are measured in $\frac{\text{kg}\cdot\text{m}}{\text{s}}$ — there is no special name for that unit.

6.2 Conservation of Momentum

Momemtum is always conserved in any closed system:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

For two objects, this becomes:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Any time two objects interact, the change in momentum of one object is equal in magnitude and opposite in direction to the change in momentum of the other object:

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

6.3 Elastic and Inelastic Collisions

There are three types of collisions:

- **Elastic** — momentum and kinetic energy are conserved. Both objects return to their original shape and move away separately. Generally:

  $$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

  $$\Sigma KE_i = \Sigma KE_f$$

  For two objects:

  $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

  $$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

- **Inelastic** — momentum is conserved, kinetic energy is lost. One or more of the objects is deformed in the collision. Generally:

  $$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

  $$\Sigma KE_i > \Sigma KE_f$$

  For two objects:

  $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

  $$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 > \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

- **Perfectly inelastic** — momentum is conserved, kinetic energy is lost. One or more objects may be deformed and the objects stick together after the collision. Generally:

  $$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

  $$\Sigma KE_i > \Sigma KE_f$$

  For two objects:

  $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

  $$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 > \frac{1}{2}(m_1 + m_2)v_f^2$$

True elastic or perfectly inelastic collisions are very rare in the real world. If we ignore friction and other small energy losses many collisions may be modeled by them.

Newton's Laws in terms of Momentum

1. Inertia:
   $$\Sigma\vec{F} = 0 \longrightarrow \vec{p} = constant$$
2. $\vec{F} = m\vec{a}$:
   $$\Delta\vec{p} = \vec{F}\Delta t$$
3. Every action has an equal and opposite reaction:
   $$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

Chapter 7 — Rotational Motion and the Law of Gravity

7.1 Measuring Ratational Motion

For an object moving in a circle with radius r through an arc length of s, the angle θ (in radians) swept by the object is:

$$\theta = \frac{s}{r}$$

The conversion between radians and degrees is:

$$\theta(\text{rad}) = \frac{\pi}{180^o}\theta(\text{deg})$$

The **angular displacement** ($\Delta\theta$) through which an object moves from θ_i to θ_f is, in rad:

$$\Delta\theta = \theta_f - \theta_i = \frac{s_f - s_i}{r} = \frac{\Delta s}{r}$$

The average **angular speed** (ω) of an object is, in $\frac{\text{rad}}{\text{s}}$, the ratio between the angular displacement and the time interval required for that displacement:

$$\omega_{avg} = \frac{\theta_f - \theta_i}{\Delta t} = \frac{\Delta\theta}{\Delta t}$$

The average **angular acceleration** (α) is, in $\frac{\text{rad}}{\text{s}^2}$:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\Delta\omega}{\Delta t}$$

For each quantity or relationship in linear motion there is a corresponding quantity or relationship in angular motion:

Linear	Angular
x	θ
v	ω
a	α
$v_f = v_i + a\Delta t$	$\omega_f = \omega_i + \alpha\Delta t$
$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$	$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$
$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$	$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$

7.2 Tangential and Centripetal Acceleration

In addition to the angular speed of an object moving with circular motion it is also possible to measure the object's **tangential speed** (v_t) or instantaneous linear speed which is measured in $\frac{\text{m}}{\text{s}}$ as follows:

$$v_t = r\omega$$

An object's **tangential acceleration** (a_t) can also be measured, in $\frac{\text{m}}{\text{s}^2}$, as follows:

$$a_t = r\alpha$$

The **centripetal acceleration** (a_c) of an object is directed toward the center of the object's rotation and has the following magnitude:

$$a_c = \frac{v_t^2}{r} = r\omega^2$$

7.3 Causes of Circular Motion

The **centripetal force** (F_c) causing a centripetal acceleration is also directed toward the center of the object's rotation, and has the following magnitude:

$$F_c = ma_c = \frac{mv_t^2}{r} = mr\omega^2$$

The centripetal force keeping planets in orbit is a **gravitational force** (F_g) and it is found with Newton's Universal Law of Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

where G is the **constant of universal gravitation** which has been determined experimentally to be

$$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

An object in a circular orbit around the Earth will satisfy the equation

$$v = \sqrt{\frac{GM_E}{r}}$$

where M_E is the mass of the Earth.

Chapter 8 — Rotational Equilibrium and Dynamics

8.1 Torque

Just as a net external force acting on an object causes linear acceleration, a net **torque** (τ) causes angular acceleration. The torque caused by a force acting on an object is:

$$\tau = rF\sin\theta$$

where F is the force causing the torque, r is the distance from the center to the point where the force acts on the object, and θ is the angle between the force and a radial line from the object's center through the point where the force is acting. Torque is measured in $\mathrm{N \cdot m}$.

The convention used by the book is that torque in a counterclockwise direction is positive and torque in a clockwise direction is negative (this corresponds to the right-hand rule). If more that one force is acting on an object the torques from each force can be added to find the net torque:

$$\tau_{net} = \sum \tau$$

8.2 Rotation and Inertia

The **center of mass** of an object is the point at which all the mass of the object can be said to be concentrated. If the object rotates freely it will rotate about the center of mass.

The **center of gravity** of an object is the point through which a gravitational force acts on the object. For most objects the center of mass and center of gravity will be the same point.

The **moment of inertia** (I) of an object is the object's resistance to changes in rotational motion about some axis. Moment of inertia in rotational motion is analogous to mass in translational motion.

Some moments of inertia for various common shapes are:

Shape	I
Point mass at a distance r from the axis	mr^2
Solid disk or cylinder of radius r about the axis	$\frac{1}{2}mr^2$
Solid sphere of radius r about its diameter	$\frac{2}{5}mr^2$
Thin spherical shell of radius r about its diameter	$\frac{2}{3}mr^2$
Thin hoop of radius r about the axis	mr^2
Thin hoop of radius r about the diameter	$\frac{1}{2}mr^2$
Thin rod of length l about its center	$\frac{1}{12}ml^2$
Thin rod of length l about its end	$\frac{1}{3}ml^2$

An object is said to be in **rotational equilibrium** when there is no net torque acting on the object. If there is also no net force acting on the object (**translational equilibrium**) then the object is in **equilibrium** (without any qualifying terms).

Type	Equation	Meaning
Translational Equilibrium	$\sum \vec{F} = 0$	The net force on the object is zero
Rotational Equilibrium	$\sum \tau = 0$	The net torque on the object is zero

8.3 Rotational Dynamics

Newton's second law can be restated for angular motion as:

$$\tau_{net} = I\alpha$$

This is parallel to the equation for translational motion as follows:

Type of Motion	Equation
Translational	$\vec{F} = m\vec{a}$
Rotational	$\tau = I\alpha$

Just as moment of inertia was analogous to mass, the **angular momentum** (L) in rotational motion is analogous to the momentum of an object in translational motion.

This is parallel to the equation for translational motion as follows:

Type of Motion	Equation
Translational	$\vec{p} = m\vec{v}$
Rotational	$L = I\omega$

The angular momemtum of an object is conserved in the absence of an external force or torque.

Rotating objects have **rotational kinetic energy** according to the following equation:

$$KE_r = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

Just as other types of mechanical energy may be conserved, rotational kinetic energy is also conserved in the absence of friction.

8.4 Simple Machines

There are six fundamental types of machines, called **simple machines**, with which any other machine can be constructed. They are levers, inclined planes, wheels, wedges, pulleys, and screws.

The main purpose of a machine is to magnify the output force of the machine compared to the input force, the ratio of these forces is called the **mechanical advantage** (MA)

of the machine. It is a unitless number according to the following equation:

$$MA = \frac{output\ force}{input\ force} = \frac{F_{out}}{F_{in}}$$

When frictional forces are accounted for, some of the output force is lost, causing less work to be done by the machine than by the original force. The ratio of work done by the machine to work put in to the machine is called the **efficiency** (*eff*) of the machine:

$$eff = \frac{W_{out}}{W_{in}}$$

If a machine is perfectly efficient ($eff = 1$) then the **ideal mechanical advantage** (*IMA*) can be found by comparing the input and output distances:

$$IMA = \frac{d_{in}}{d_{out}}$$

This leads to another way to find the efficiency of the machine as well:

$$eff = \frac{MA}{IMA}$$

Variables and Notation

SI Prefixes

Prefix	Mult.	Abb.	Prefix	Mult.	Abb.
yocto-	10^{-24}	y	yotta-	10^{24}	Y
zepto-	10^{-21}	z	zetta-	10^{21}	Z
atto-	10^{-18}	a	exa-	10^{18}	E
femto-	10^{-15}	f	peta-	10^{15}	P
pico-	10^{-12}	p	tera-	10^{12}	T
nano-	10^{-9}	n	giga-	10^{9}	G
micro-	10^{-6}	μ	mega-	10^{6}	M
milli-	10^{-3}	m	kilo-	10^{3}	k
centi-	10^{-2}	c	hecto-	10^{2}	h
deci-	10^{-1}	d	deka-	10^{1}	da

Notation

Notation	Description
$\vec{x}$	Vector
x	Scalar, or the magnitude of $\vec{x}$
$\lvert\vec{x}\rvert$	The absolute value or magnitude of $\vec{x}$
Δx	Change in x
$\sum x$	Sum of all x
Πx	Product of all x
x_i	Initial value of x
x_f	Final value of x
$\hat{x}$	Unit vector in the direction of x
$A \longrightarrow B$	A implies B
$A \propto B$	A is proportional to B
$A \gg B$	A is much larger than B

Units

Symbol	Unit	Quantity	Composition
kg	kilogram	Mass	SI base unit
m	meter	Length	SI base unit
s	second	Time	SI base unit
A	ampere	Electric current	SI base unit
cd	candela	Luminous intensity	SI base unit
K	kelvin	Temperature	SI base unit
mol	mole	Amount	SI base unit
Ω	ohm	Resistance	$\frac{\mathrm{V}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A^2}}$
C	coulomb	Charge	$\mathrm{A \cdot s}$
F	farad	Capacitance	$\frac{\mathrm{C}}{\mathrm{V}}$ or $\frac{\mathrm{s^4 \cdot A^2}}{\mathrm{m^2 \cdot kg}}$
H	henry	Inductance	$\frac{\mathrm{V \cdot s}}{\mathrm{A}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{A^2 \cdot s^2}}$
Hz	hertz	Frequency	$\mathrm{s^{-1}}$
J	joule	Energy	$\mathrm{N \cdot m}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$
N	newton	Force	$\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$
rad	radian	Angle	$\frac{\mathrm{m}}{\mathrm{m}}$ or 1
T	tesla	Magnetic field	$\frac{\mathrm{N}}{\mathrm{A \cdot m}}$
V	volt	Electric potential	$\frac{\mathrm{J}}{\mathrm{C}}$ or $\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^3 \cdot A}}$
W	watt	Power	$\frac{\mathrm{J}}{\mathrm{s}}$ or $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}$
Wb	weber	Magnetic flux	$\mathrm{V \cdot s}$ or $\frac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot A}}$

Greek Alphabet

Name	Maj.	Min.	Name	Maj.	Min.
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π or ϖ
Epsilon	E	ϵ or ε	Rho	P	ρ or ϱ
Zeta	Z	ζ	Sigma	Σ	σ or ς
Eta	H	η	Tau	T	τ
Theta	Θ	θ or ϑ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ or φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Variables

Variable	Description	Units
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ	Angular position	rad
θ_c	Critical angle	^{o} (degrees)
θ_i	Incident angle	^{o} (degrees)
θ_r	Refracted angle	^{o} (degrees)
θ'	Reflected angle	^{o} (degrees)
$\Delta\theta$	Angular displacement	rad
τ	Torque	$\text{N}\cdot\text{m}$
ω	Angular speed	$\frac{\text{rad}}{\text{s}}$
μ	Coefficient of friction	(unitless)
μ_k	Coefficient of kinetic friction	(unitless)
μ_s	Coefficient of static friction	(unitless)
$\vec{a}$	Acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_c$	Centripetal acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_g$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_t$	Tangential acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_x$	Acceleration in the x direction	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_y$	Acceleration in the y direction	$\frac{\text{m}}{\text{s}^2}$
A	Area	m^2
B	Magnetic field strength	T
C	Capacitance	F
$\vec{d}$	Displacement	m
$d\sin\theta$	lever arm	m
$\vec{d}_x$ or Δx	Displacement in the x direction	m
$\vec{d}_y$ or Δy	Displacement in the y direction	m
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
f	Focal length	m
$\vec{F}$	Force	N
$\vec{F}_c$	Centripetal force	N
$\vec{F}_{electric}$	Electrical force	N
$\vec{F}_g$	Gravitational force	N
$\vec{F}_k$	Kinetic frictional force	N
$\vec{F}_{magnetic}$	Magnetic force	N
$\vec{F}_n$	Normal force	N
$\vec{F}_s$	Static frictional force	N
$\vec{F}\Delta t$	Impulse	$\text{N}\cdot\text{s}$ or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$
$\vec{g}$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$

Variable	Description	Units
h	Object height	m
h'	Image height	m
I	Current	A
I	Moment of inertia	$\text{kg}\cdot\text{m}^2$
KE or K	Kinetic energy	J
KE_{rot}	Rotational kinetic energy	J
L	Angular Momentum	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$
L	Self-inductance	H
m	Mass	kg
M	Magnification	(unitless)
M	Mutual inductance	H
MA	Mechanical Advantage	(unitless)
ME	Mechanical Energy	J
n	Index of refraction	(unitless)
p	Object distance	m
$\vec{p}$	Momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
P	Power	W
PE or U	Potential Energy	J
$PE_{elastic}$	Elastic potential energy	J
$PE_{electric}$	Electrical potential energy	J
PE_g	Gravitational potential energy	J
q	Image distance	m
q or Q	Charge	C
Q	Heat, Entropy	J
R	Radius of curvature	m
R	Resistance	Ω
s	Arc length	m
t	Time	s
Δt	Time interval	s
$\vec{v}$	Velocity	$\frac{\text{m}}{\text{s}}$
v_t	Tangential speed	$\frac{\text{m}}{\text{s}}$
$\vec{v}_x$	Velocity in the x direction	$\frac{\text{m}}{\text{s}}$
$\vec{v}_y$	Velocity in the y direction	$\frac{\text{m}}{\text{s}}$
V	Electric potential	V
ΔV	Electric potential difference	V
V	Volume	m^3
W	Work	J
x or y	Position	m

Constants

Symbol	Name	Established Value	Value Used
ϵ_0	Permittivity of a vacuum	$8.854\ 187\ 817 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}$	$8.85 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}$
ϕ	Golden ratio	1.618 033 988 749 894 848 20	
π	Archimedes' constant	3.141 592 653 589 793 238 46	
g, a_g	Gravitational acceleration constant	$9.79171\ \frac{\mathrm{m}}{\mathrm{s}^2}$ (varies by location)	$9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$
c	Speed of light in a vacuum	$2.997\ 924\ 58 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$ (exact)	$3.00 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$
e	Natural logarithmic base	2.718 281 828 459 045 235 36	
e^-	Elementary charge	$1.602\ 177\ 33 \times 10^{19}$ C	1.60×10^{19} C
G	Gravitational constant	$6.672\ 59 \times 10^{-11}\ \frac{\mathrm{N \cdot m}^2}{\mathrm{kg}^2}$	$6.67 \times 10^{-11}\ \frac{\mathrm{N \cdot m}^2}{\mathrm{kg}^2}$
k_C	Coulomb's constant	$8.987\ 551\ 788 \times 10^{9}\ \frac{\mathrm{N \cdot m}^2}{\mathrm{C}^2}$	$8.99 \times 10^{9}\ \frac{\mathrm{N \cdot m}^2}{\mathrm{C}^2}$
N_A	Avogadro's constant	$6.022\ 141\ 5 \times 10^{23}\ \mathrm{mol}^{-1}$	

Astronomical Data

Symbol	Object	Mean Radius	Mass	Mean Orbit Radius	Orbital Period
☽	Moon	1.74×10^{6} m	7.36×10^{22} kg	3.84×10^{8} m	2.36×10^{6} s
☉	Sun	6.96×10^{8} m	1.99×10^{30} kg	—	—
☿	Mercury	2.43×10^{6} m	3.18×10^{23} kg	5.79×10^{10} m	7.60×10^{6} s
♀	Venus	6.06×10^{6} m	4.88×10^{24} kg	1.08×10^{11} m	1.94×10^{7} s
♁	Earth	6.37×10^{6} m	5.98×10^{24} kg	1.496×10^{11} m	3.156×10^{7} s
♂	Mars	3.37×10^{6} m	6.42×10^{23} kg	2.28×10^{11} m	5.94×10^{7} s
	Ceres[1]	4.71×10^{5} m	9.5×10^{20} kg	4.14×10^{11} m	1.45×10^{8} s
♃	Jupiter	6.99×10^{7} m	1.90×10^{27} kg	7.78×10^{11} m	3.74×10^{8} s
♄	Saturn	5.85×10^{7} m	5.68×10^{26} kg	1.43×10^{12} m	9.35×10^{8} s
♅	Uranus	2.33×10^{7} m	8.68×10^{25} kg	2.87×10^{12} m	2.64×10^{9} s
♆	Neptune	2.21×10^{7} m	1.03×10^{26} kg	4.50×10^{12} m	5.22×10^{9} s
♇	Pluto[1]	1.15×10^{6} m	1.31×10^{22} kg	5.91×10^{12} m	7.82×10^{9} s
	Eris[2,1]	2.4×10^{6} m	1.5×10^{22} kg	1.01×10^{13} m	1.75×10^{10} s

[1] Ceres, Pluto, and Eris are classified as "Dwarf Planets" by the IAU
[2] Eris was formerly known as 2003 UB_{313}

Mathematics Review for Physics

This is a summary of the most important parts of mathematics as we will use them in a physics class. There are numerous parts that are completely omitted, others are greatly abridged. Do not assume that this is a complete coverage of any of these topics.

Algebra

Fundamental properties of algebra

$a+b=b+a$	Commutative law for addition
$(a+b)+c=a+(b+c)$	Associative law for addition
$a+0=0+a=a$	Identity law for addition
$a+(-a)=(-a)+a=0$	Inverse law for addition
$ab=ba$	Commutative law for multiplication
$(ab)c=a(bc)$	Associative law for multiplication
$(a)(1)=(1)(a)=a$	Identity law for multiplication
$a\frac{1}{a}=\frac{1}{a}a=1$	Inverse law for multiplication
$a(b+c)=ab+ac$	Distributive law

Exponents

$$(ab)^n = a^n b^n \qquad (a/b)^n = a^n/b^n$$
$$a^n a^m = a^{n+m} \qquad 0^n = 0$$
$$a^n/a^m = a^{n-m} \qquad a^0 = 1$$
$$(a^n)^m = a^{(mn)} \qquad 0^0 = 1 \text{ (by definition)}$$

Logarithms

$$x = a^y \longrightarrow y = \log_a x$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^n) = n\log_a x$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a} = (\log_b x)(\log_a b)$$

Binomial Expansions

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a^i b^{n-i}$$

Quadratic formula

For equations of the form $ax^2+bx+c=0$ the solutions are:

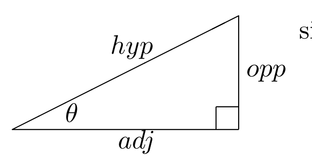
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Geometry

Shape	Area	Volume
Triangle	$A=\frac{1}{2}bh$	—
Rectangle	$A=lw$	—
Circle	$A=\pi r^2$	—
Rectangular prism	$A=2(lw+lh+hw)$	$V=lwh$
Sphere	$A=4\pi r^2$	$V=\frac{4}{3}\pi r^3$
Cylinder	$A=2\pi rh+2\pi r^2$	$V=\pi r^2 h$
Cone	$A=\pi r\sqrt{r^2+h^2}+\pi r^2$	$V=\frac{1}{3}\pi r^2 h$

Trigonometry

In physics only a small subset of what is covered in a trigonometry class is likely to be used, in particular sine, cosine, and tangent are useful, as are their inverse functions. As a reminder, the relationships between those functions and the sides of a right triangle are summarized as follows:



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

The inverse functions are only defined over a limited range. The $\tan^{-1} x$ function will yield a value in the range $-90° < \theta < 90°$, $\sin^{-1} x$ will be in $-90° \le \theta \le 90°$, and $\cos^{-1} x$ will yield one in $0° \le \theta \le 180°$. Care must be taken to ensure that the result given by a calculator is in the correct quadrant, if it is not then an appropriate correction must be made.

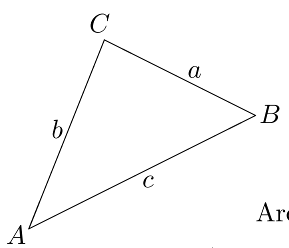
Degrees	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

$$\sin^2\theta + \cos^2\theta = 1 \qquad 2\sin\theta\cos\theta = \sin(2\theta)$$

Trigonometric functions in terms of each other

$\sin\theta =$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$
$\cos\theta =$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$
$\tan\theta =$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$
$\sec\theta =$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$
$\cot\theta =$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$
$\sin\theta =$	$\frac{1}{\csc\theta}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\frac{1}{\sqrt{\csc^2\theta-1}}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\csc\theta$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\sqrt{\csc^2\theta-1}$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Law of sines, law of cosines, area of a triangle



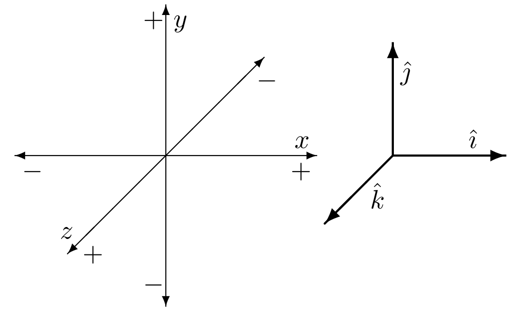
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\text{let } s = \tfrac{1}{2}(a+b+c)$$
$$\text{Area}= \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area}= \tfrac{1}{2}bc\sin A = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}ab\sin C$$

Vectors

A **vector** is a quantity with both magnitude and direction, such as displacement or velocity. Your textbook indicates a vector in bold-face type as **V** and in class we have been using $\vec{V}$. Both notations are equivalent.

A **scalar** is a quantity with only magnitude. This can either be a quantity that is directionless such as time or mass, or it can be the magnitude of a vector quantity such as speed or distance traveled. Your textbook indicates a scalar in italic type as V, in class we have not done anything to distinguish a scalar quantity. The magnitude of $\vec{V}$ is written as V or $|\vec{V}|$.

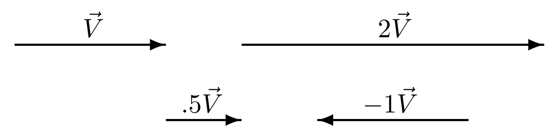
A **unit vector** is a vector with magnitude 1 (a dimensionless constant) pointing in some significant direction. A unit vector pointing in the direction of the vector $\vec{V}$ is indicated as $\hat{V}$ and would commonly be called V-hat. Any vector can be normalized into a unit vector by dividing it by its magnitude, giving $\hat{V} = \frac{\vec{V}}{V}$. Three special unit vectors, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are introduced with chapter 3. They point in the directions of the positive x, y, and z axes, respectively (as shown below).



Vectors can be added to other vectors of the same dimension (i.e. a velocity vector can be added to another velocity vector, but not to a force vector). The sum of all vectors to be added is called the **resultant** and is equivalent to all of the vectors combined.

Multiplying Vectors

Any vector can be multiplied by any scalar, this has the effect of changing the magnitude of the vector but not its direction (with the exception that multiplying a vector by a negative scalar will reverse the direction of the vector). As an example, multiplying a vector $\vec{V}$ by several scalars would give:



In addition to scalar multiplication there are also two ways to multiply vectors by other vectors. They will not be directly used in class but being familiar with them may help to understand how some physics equations are derived. The first, the **dot product** of vectors $\vec{V}_1$ and $\vec{V}_2$, represented as $\vec{V}_1 \cdot \vec{V}_2$ measures the tendency of the two vectors to point in the same direction. If the angle between the two vectors is θ the dot product yields a scalar value as

$$\vec{V}_1 \cdot \vec{V}_2 = V_1 V_2 \cos\theta$$

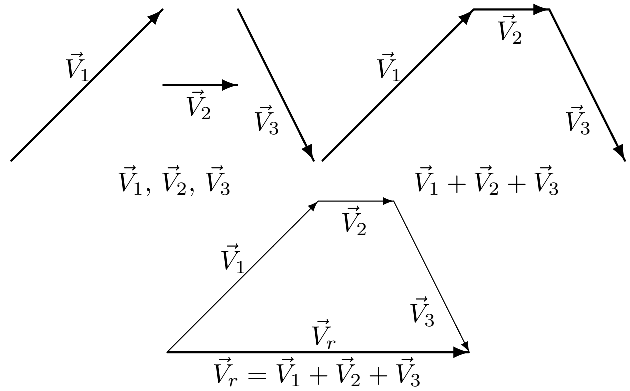
The second method of multiplying two vectors, the **cross product**, (represented as $\vec{V}_1 \times \vec{V}_2$) measures the tendency of vectors to be perpendicular to each other. It yields a third vector perpendicular to the two original vectors with magnitude

$$|\vec{V}_1 \times \vec{V}_2| = V_1 V_2 \sin\theta$$

The direction of the cross product is perpendicular to the two vectors being crossed and is found with the right-hand rule — point the fingers of your right hand in the direction of the first vector, curl them toward the second vector, and the cross product will be in the direction of your thumb.

Adding Vectors Graphically

The sum of any number of vectors can be found by drawing them head-to-tail to scale and in proper orientation then drawing the resultant vector from the tail of the first vector to the point of the last one. If the vectors were drawn accurately then the magnitude and direction of the resultant can be measured with a ruler and protractor. In the example below the vectors $\vec{V}_1$, $\vec{V}_2$, and $\vec{V}_3$ are added to yield $\vec{V}_r$

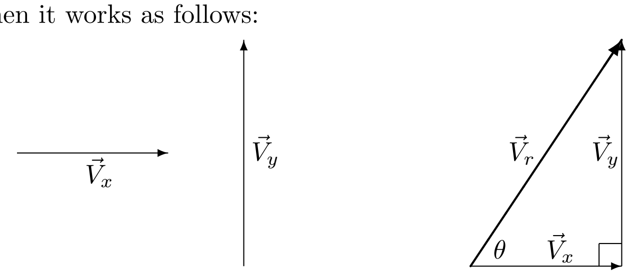


Adding Parallel Vectors

Any number of parallel vectors can be directly added by adding their magnitudes if one direction is chosen as positive and vectors in the opposite direction are assigned a negative magnitude for the purposes of adding them. The sum of the magnitudes will be the magnitude of the resultant vector in the positive direction, if the sum is negative then the resultant will point in the negative direction.

Adding Perpendicular Vectors

Perpendicular vectors can be added by drawing them as a right triangle and then finding the magnitude and direction of the hypotenuse (the resultant) through trigonometry and the Pythagorean theorem. If $\vec{V}_r = \vec{V}_x + \vec{V}_y$ and $\vec{V}_x \perp \vec{V}_y$ then it works as follows:



Since the two vectors to be added and the resultant form a right triangle with the resultant as the hypotenuse the Pythagorean theorem applies giving

$$V_r = |\vec{V}_r| = \sqrt{V_x^2 + V_y^2}$$

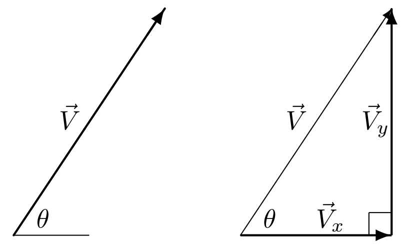
The angle θ can be found by taking the inverse tangent of the ratio between the magnitudes of the vertical and horizontal vectors, thus

$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

As was mentioned above, care must be taken to ensure that the angle given by the calculator is in the appropriate quadrant for the problem, this can be checked by looking at the diagram drawn to solve the problem and verifying that the answer points in the direction expected, if not then make an appropriate correction.

Resolving a Vector Into Components

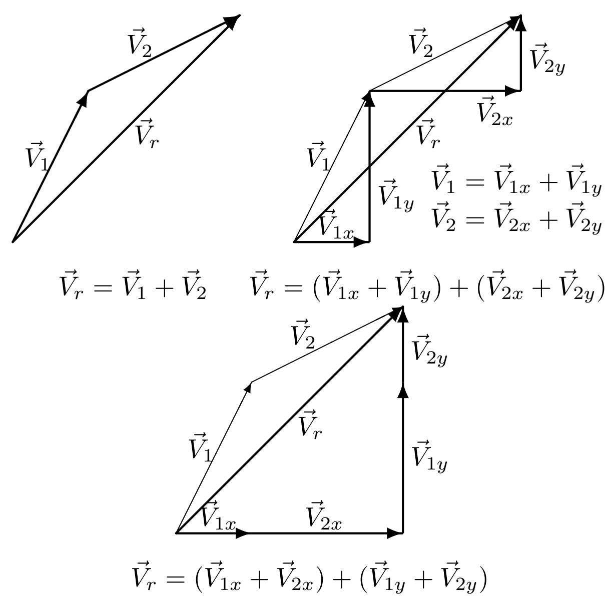
Just two perpendicular vectors can be added to find a single resultant, any single vector $\vec{V}$ can be resolved into two perpendicular **component vectors** $\vec{V}_x$ and $\vec{V}_y$ so that $\vec{V} = \vec{V}_x + \vec{V}_y$.



As the vector and its components can be drawn as a right triangle the ratios of the sides can be found with trigonometry. Since $\sin\theta = \frac{V_y}{V}$ and $\cos\theta = \frac{V_x}{V}$ it follows that $V_x = V\cos\theta$ and $V_y = V\sin\theta$ or in a vector form, $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$. (This is actually an application of the dot product, $\vec{V}_x = (\vec{V}\cdot\hat{\imath})\hat{\imath}$ and $\vec{V}_y = (\vec{V}\cdot\hat{\jmath})\hat{\jmath}$, but it is not necessary to know that for this class)

Adding Any Two Vectors Algebraically

Only vectors with the same direction can be directly added, so if vectors pointing in multiple directions must be added they must first be broken down into their components, then the components are added and resolved into a single resultant vector — if in two dimensions $\vec{V}_r = \vec{V}_1 + \vec{V}_2$ then



Once the sums of the component vectors in each direction have been found the resultant can be found from them just as an other perpendicular vectors may be added. Since from the last figure $\vec{V}_r = (\vec{V}_{1x}+\vec{V}_{2x})+(\vec{V}_{1y}+\vec{V}_{2y})$ and it was previously established that $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$ it follows that

$$\vec{V}_r = (V_1\cos\theta_1 + V_2\cos\theta_2)\,\hat{\imath} + (V_1\sin\theta_1 + V_2\sin\theta_2)\,\hat{\jmath}$$

and

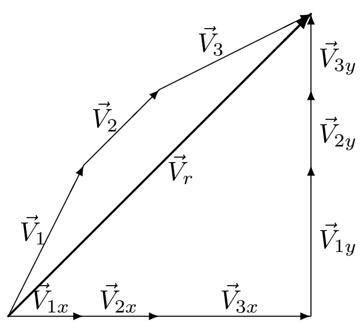
$$\begin{aligned} V_r &= \sqrt{(V_{1x}+V_{2x})^2 + (V_{1y}+V_{2y})^2} \\ &= \sqrt{(V_1\cos\theta_1 + V_2\cos\theta_2)^2 + (V_1\sin\theta_1 + V_2\sin\theta_2)^2} \end{aligned}$$

with the direction of the resultant vector $\vec{V}_r$, θ_r, being found with

$$\theta_r = \tan^{-1}\frac{V_{1y}+V_{2y}}{V_{1x}+V_{2x}} = \tan^{-1}\frac{V_1\sin\theta_1 + V_2\sin\theta_2}{V_1\cos\theta_1 + V_2\cos\theta_2}$$

Adding Any Number of Vectors Algebraically

For a total of n vectors $\vec{V}_i$ being added with magnitudes V_i and directions θ_i the magnitude and direction are:



$$\vec{V}_r = \left(\sum_{i=1}^{n} V_i\cos\theta_i\right)\hat{\imath} + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)\hat{\jmath}$$

$$V_r = \sqrt{\left(\sum_{i=1}^{n} V_i\cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)^2}$$

$$\theta_r = \tan^{-1}\frac{\sum_{i=1}^{n} V_i\sin\theta_i}{\sum_{i=1}^{n} V_i\cos\theta_i}$$

(The figure shows only three vectors but this method will work with any number of them so long as proper care is taken to ensure that all angles are measured the same way and that the resultant direction is in the proper quadrant.)

Calculus

Although this course is based on algebra and not calculus, it is sometimes useful to know some of the properties of derivatives and integrals. If you have not yet learned calculus it is safe to skip this section.

Derivatives

The **derivative** of a function $f(t)$ with respect to t is a function equal to the slope of a graph of $f(t)$ vs. t at every point, assuming that slope exists. There are several ways to indicate that derivative, including:

$$\frac{\mathrm{d}}{\mathrm{d}t}f(t)$$
$$f'(t)$$
$$\dot{f}(t)$$

The first one from that list is unambiguous as to the independent variable, the others assume that there is only one variable, or in the case of the third one that the derivative is taken with respect to time. Some common derivatives are:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}t^n &= nt^{n-1} \\ \frac{\mathrm{d}}{\mathrm{d}t}\sin t &= \cos t \\ \frac{\mathrm{d}}{\mathrm{d}t}\cos t &= -\sin t \\ \frac{\mathrm{d}}{\mathrm{d}t}e^t &= e^t \end{aligned}$$

If the function is a compound function then there are a few useful rules to find its derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}cf(t) = c\frac{\mathrm{d}}{\mathrm{d}t}f(t) \qquad c \text{ constant for all } t$$
$$\frac{\mathrm{d}}{\mathrm{d}t}[f(t)+g(t)] = \frac{\mathrm{d}}{\mathrm{d}t}f(t) + \frac{\mathrm{d}}{\mathrm{d}t}g(t)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}f(u) = \frac{\mathrm{d}}{\mathrm{d}t}u\frac{\mathrm{d}}{\mathrm{d}u}f(u)$$

Integrals

The **integral**, or **antiderivative** of a function $f(t)$ with respect to t is a function equal to the the area under a graph of $f(t)$ vs. t at every point plus a constant, assuming that $f(t)$ is continuous. Integrals can be either indefinite or definite. An indefinite integral is indicated as:

$$\int f(t)\,\mathrm{d}t$$

while a definite integral would be indicated as

$$\int_a^b f(t)\,\mathrm{d}t$$

to show that it is evaluated from $t = a$ to $t = b$, as in:

$$\int_a^b f(t)\,\mathrm{d}t = \int f(t)\,\mathrm{d}t\Big|_{t=a}^{t=b} = \int f(t)\,\mathrm{d}t\Big|_{t=b} - \int f(t)\,\mathrm{d}t\Big|_{t=a}$$

Some common integrals are:

$$\int t^n\,\mathrm{d}t = \frac{t^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int t^{-1}\,\mathrm{d}t = \ln t + C$$

$$\int \sin t\,\mathrm{d}t = -\cos t + C$$

$$\int \cos t\,\mathrm{d}t = \sin t + C$$

$$\int e^t\,\mathrm{d}t = e^t + C$$

There are rules to reduce integrals of some compound functions to simpler forms (there is no general rule to reduce the integral of the product of two functions):

$$\int cf(t)\,\mathrm{d}t = c\int f(t)\,\mathrm{d}t \qquad c \text{ constant for all } t$$

$$\int [f(t)+g(t)]\,\mathrm{d}t = \int f(t)\,\mathrm{d}t + \int g(t)\,\mathrm{d}t$$

Series Expansions

Some functions are difficult to work with in their normal forms, but once converted to their series expansion can be manipulated easily. Some common series expansions are:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{i=0}^{\infty}(-1)^i \frac{x^{2i+1}}{(2i+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{i=0}^{\infty}(-1)^i \frac{x^{2i}}{(2i)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{(i)!}$$

Physics Using Calculus

In our treatment of mechanics the majority of the equations that have been covered in the class have been either approximations or special cases where the acceleration or force applied have been held constant. This is required because the class is based on algebra and to do otherwise would require the use of calculus.

None of what is presented here is a required part of the class, it is here to show how to handle cases outside the usual approximations and simplifications. Feel free to skip this section, none of it will appear as a required part of any assignment or test in this class.

Notation

Because the letter 'd' is used as a part of the notation of calculus the variable 'r' is often used to represent the position vector of the object being studied. (Other authors use 's' for the spatial position or generalize 'x' to two or more dimensions.) In a vector form that would become $\vec{r}$ to give both the magnitude and direction of the position. It is assumed that the position, velocity, acceleration, etc. can be expressed as a function of time as $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ but the dependency on time is usually implied and not shown explicitly.

Translational Motion

Many situations will require an acceleration that is not constant. Everything learned about translational motion so far used the simplification that the acceleration remained constant, with calculus we can let the acceleration be any function of time.

Since velocity is the slope of a graph of position vs. time at any point, and acceleration is the slope of velocity vs. time these can be expressed mathematically as derivatives:

$$\vec{v} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}$$

$$\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}$$

The reverse of this relationship is that the displacement of an object is equal to the area under a velocity vs. time graph and velocity is equal to the area under an acceleration vs. time graph. Mathematically this can be expressed as integrals:

$$\vec{v} = \int \vec{a}\,\mathrm{d}t$$

$$\vec{r} = \int \vec{v}\,\mathrm{d}t = \iint \vec{a}\,\mathrm{d}t^2$$

Using these relationships we can derive the main equations of motion that were introduced by starting with the integral of a constant velocity $\vec{a}(t) = \vec{a}$ as

$$\int_{t_i}^{t_f} \vec{a}\,\mathrm{d}t = \vec{a}\Delta t + C$$

The constant of integration, C, can be shown to be the initial velocity, $\vec{v}_i$, so the entire expression for the final velocity becomes

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

or

$$\vec{v}_f(t) = \vec{v}_i + \vec{a}t$$

Similarly, integrating that expression with respect to time gives an expression for position:

$$\int_{t_i}^{t_f} (\vec{v}_i + \vec{a}t)\,\mathrm{d}t = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2 + C$$

Once again the constant of integration, C, can be shown to be the initial position, $\vec{r}_i$, yielding an expression for position vs. time

$$\vec{r}_f(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

While calculus can be used to derive the algebraic forms of the equations the reverse is not true. Algebra can handle a subset of physics and only at the expense of needing to remember separate equations for various special cases. With calculus the few definitions shown above will suffice to predict any linear motion.

Work and Power

The work done on an object is the integral of dot product of the force applied with the path the object takes, integrated as a contour integral over the path.

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The power developed is the time rate of change of the work done, or the derivative of the work with respect to time.

$$P = \frac{\mathrm{d}}{\mathrm{d}t}W$$

Momentum

Newton's second law of motion changes from the familiar $\vec{F} = m\vec{a}$ to the statement that the force applied to an object is the time derivative of the object's momentum.

$$\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{p}$$

Similarly, the impulse delivered to an object is the integral of the force applied in the direction parallel to the motion with respect to time.

$$\Delta\vec{p} = \int \vec{F}\,\mathrm{d}t$$

Rotational Motion

The equations for rotational motion are very similar to those for translational motion with appropriate variable substitutions. First, the angular velocity is the time derivative of the angular position.

$$\vec{\omega} = \frac{\mathrm{d}\vec{\theta}}{\mathrm{d}t}$$

The angular acceleration is the time derivative of the angular velocity or the second time derivative of the angular position.

$$\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\theta}}{\mathrm{d}t^2}$$

The angular velocity is also the integral of the angular acceleration with respect to time.

$$\vec{\omega} = \int \vec{\alpha}\,\mathrm{d}t$$

The angular position is the integral of the angular velocity with respect to time or the second integral of the angular acceleration with respect to time.

$$\vec{\theta} = \int \vec{\omega}\,\mathrm{d}t = \iint \vec{\alpha}\,\mathrm{d}t^2$$

The torque exerted on an object is the cross product of the radius vector from the axis of rotation to the point of action with the force applied.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The moment of inertia of any object is the integral of the square of the distance from the axis of rotation to each element of the mass over the entire mass of the object.

$$I = \int r^2\,\mathrm{d}m$$

The torque applied to an object is the time derivative of the object's angular momentum.

$$\vec{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

The integral of the torque with respect to time is the angular momentum of the object.

$$\vec{L} = \int \vec{\tau}\,\mathrm{d}t$$

Data Analysis

Comparative Measures

Percent Error

When experiments are conducted where there is a calculated result (or other known value) against which the observed values will be compared the percent error between the observed and calculated values can be found with

$$\textit{percent error} = \left| \frac{\textit{observed value} - \textit{accepted value}}{\textit{accepted value}} \right| \times 100\%$$

Percent Difference

When experiments involve a comparison between two experimentally determined values where neither is regarded as correct then rather than the percent error the percent difference can be calculated. Instead of dividing by the accepted value the difference is divided by the mean of the values being compared. For any two values, a and b, the percent difference is

$$\textit{percent difference} = \left| \frac{a - b}{\frac{1}{2}(a + b)} \right| \times 100\%$$

Dimensional Analysis and Units on Constants

Consider a mass vibrating on the end of a spring. The equation relating the mass to the frequency on graph might look something like

$$y = \frac{3.658}{x^2} - 1.62$$

This isn't what we're looking for, a first pass would be to change the x and y variables into f and m for frequency and mass, this would then give

$$m = \frac{3.658}{f^2} - 1.62$$

This is progress but there's still a long way to go. Mass is measured in kg and frequency is measured in s^{-1}, since these variables represent the entire measurement (including the units) and not just the numeric portion they do not need to be labeled, but the constants in the equation do need to have the correct units applied.

To find the units it helps to first re-write the equation using just the units, letting some variable such as u (or u_1, u_2, u_3, etc.) stand for the unknown units. The example equation would then become

$$\mathrm{kg} = \frac{u_1}{(\mathrm{s}^{-1})^2} + u_2$$

Since any quantities being added or subtracted must have the same units the equation can be split into two equations

$$\mathrm{kg} = \frac{u_1}{(\mathrm{s}^{-1})^2} \quad \text{and} \quad \mathrm{kg} = u_2$$

The next step is to solve for u_1 and u_2, in this case $u_2 = \mathrm{kg}$ is immediately obvious, u_1 will take a bit more effort. A first simplification yields

$$\mathrm{kg} = \frac{u_1}{\mathrm{s}^{-2}}$$

then multiplying both sides by s^{-2} gives

$$(\mathrm{s}^{-2})(\mathrm{kg}) = \left(\frac{u_1}{\mathrm{s}^{-2}}\right)(\mathrm{s}^{-2})$$

which finally simplifies and solves to

$$u_1 = \mathrm{kg} \cdot \mathrm{s}^{-2}$$

Inserting the units into the original equation finally yields

$$m = \frac{3.658\ \mathrm{kg} \cdot \mathrm{s}^{-2}}{f^2} - 1.62\ \mathrm{kg}$$

which is the final equation with all of the units in place. Checking by choosing a frequency and carrying out the calculations in the equation will show that it is dimensionally consistent and yields a result in the units for mass (kg) as expected.

Midterm Exam Description

- 99 Multiple Choice Items (31 problems, 68 questions)
- Calculators will **not** be permitted
- Many test items contain numbers, but do not require calculations to solve. Before plugging values into an equation, look at the possible responses. The correct response may be obvious due to significant figures, order of magnitude or simple common sense.
- Since all test items are worth one point, you should first complete the items that you are most sure of and then go through the exam a second time. You should make sure that you answer every question on the exam.
- Exam items are from the following categories:
  - **Chapter 1 — The Science of Physics**
    units of measurement
    significant figures
  - **Chapter 2 — Motion in One Dimension**
    displacement and velocity
    acceleration
    falling objects
    graphs of motion
  - **Chapter 3 — Two-Dimensional Motion and Vectors**
    vectors and vector operations
    projectiles
    relative motion
  - **Chapter 4 — Forces and the Laws of Motion**
    Newton's 1st Law
    Net forces
    Newton's 2nd Law
    Newton's 3rd Law
    Friction
  - **Chapter 5 — Work and Energy**
    work
    types of energy
    conservation of energy
    power
  - **Chapter 6 — Momentum and Collisions**
    momentum and impulse
    conservation of momentum
    types of collisions
  - **Chapter 7 — Rotational Motion and the Law of Gravity**
    measuring rotational motion
    rotational kinematics equations
    centripetal force and acceleration
    gravitational forces

Chapter 8 will not be included on the first semester exam.

Semester Exam Equation Sheet

This is a slightly reduced copy of the equation sheet that you will receive with the semester exam.

Honors Physics – 1st Semester Final
Constants and Equations

Kinematics
Linear

$$d = v_{av}\Delta t$$

$$d = \tfrac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$d = v_i\Delta t + \tfrac{1}{2}a\left(\Delta t\right)^2$$

$$v_f = v_i + a\Delta t$$

$$v_f^2 = v_i^2 + 2ad$$

$$d_x = v_x\Delta t$$

$$d_y = v_y\Delta t + \tfrac{1}{2}a_y\left(\Delta t\right)^2$$

$$v_y = v\sin\theta$$

$$v_x = v\cos\theta$$

Kinematics
Rotational

$$\theta = \frac{d}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a}{r}$$

$$\theta = \omega_{av}\Delta t$$

$$\theta = \omega_i\Delta t + \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

$$\theta = \tfrac{1}{2}\left(\omega_i + \omega_f\right)\Delta t$$

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

Dynamics
Linear

$$\sum\vec{F} = m\vec{a}$$

$$F_g = mg$$

$$F_\perp = F_g\cos\theta$$

$$F_\parallel = F_g\sin\theta$$

$$F_f = \mu F_N$$

$$\mu = \tan\theta$$

$$F_{spring} = -kx$$

$$F_{grav} = G\frac{m_1m_2}{r^2}$$

Dynamics
Circular

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F_c = m\frac{v^2}{r} = m\frac{4\pi^2 r}{T^2}$$

$$T = \tfrac{1}{f}$$

Dynamics
Rotational

$$\sum\tau = I\alpha$$

$$I_{\text{point mass}} = mr^2$$

Work, Power, Energy
Linear

$$W = Fd\cos\theta$$

$$PE_g = mgh$$

$$PE_{elastic} = \tfrac{1}{2}kx^2$$

$$KE = \tfrac{1}{2}mv^2$$

$$\Delta PE + \Delta KE + W = 0$$

$$P = \frac{W}{\Delta t} = Fv$$

Work, Power, Energy
Rotational

$$W = \tau\theta$$

$$KE = \tfrac{1}{2}I\omega^2$$

$$P = \tau\omega$$

Momentum
Linear

$$P = mv$$

$$m_1v_{1i} + m_2v_{2f} = m_1v_{1f} + m_2v_{2f}$$

$$F\Delta t = m\Delta v$$

Rotational

$$L = I\omega$$

Constants

$$g = 10.\ \tfrac{m}{s^2}$$

$$G = 6.67\times10^{-11}\ \tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

$$c = 3.00\times10^{8}\ \tfrac{\text{m}}{\text{s}}$$

$$K = 9.00\times10^{9}\ \tfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}$$

Note that some equations may be slightly different than the ones we have used in class, the sheet that you receive will have these forms of the equations, it will be up to you to know how to use them or to know other forms that you are able to use.

Mixed Review Exercises

Chapter 1

HOLT PHYSICS

Mixed Review

The Science of Physics

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	m
10^{-3}	milli-	m
10^{-2}	centi-	c

Power	Prefix	Abbreviation
10^{-1}	deci-	d
10^{1}	deka-	da
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

1. Convert the following measurements to the units specified.
   a. 2.5 days to seconds __________
   b. 35 km to millimeters __________
   c. 43 cm to kilometers __________
   d. 22 mg to kilograms __________
   e. 671 kg to micrograms __________
   f. 8.76×10^{7} mW to gigawatts __________
   g. 1.753×10^{-13} s to picoseconds __________
2. According to the rules given in Chapter 1 of your textbook, how many significant figures are there in the following measurements?
   a. 0.0845 kg __________
   b. 37.00 h __________
   c. 8 630 000.000 mi __________
   d. 0.000 000 0217 g __________
   e. 750 in. __________
   f. 0.5003 s __________

Chapter 1

HOLT PHYSICS

Mixed Review *continued*

3. **Without calculating the result,** find the number of significant figures in the following products and quotients.
   a. 0.005032×4.0009 __________
   b. $0.0080750 \div 10.037$ __________
   c. $(3.52 \times 10^{-11}) \times (7.823 \times 10^{11})$ __________
4. Calculate $a + b$, $a - b$, $a \times b$, and $a \div b$ with the correct number of significant figures using the following numbers.
   a. $a = 0.005\ 078$; $b = 1.0003$

      $a + b =$ __________ $a - b =$ __________

      $a \times b =$ __________ $a \div b =$ __________
   b. $a = 4.231\ 19 \times 10^{7}$; $b = 3.654 \times 10^{6}$

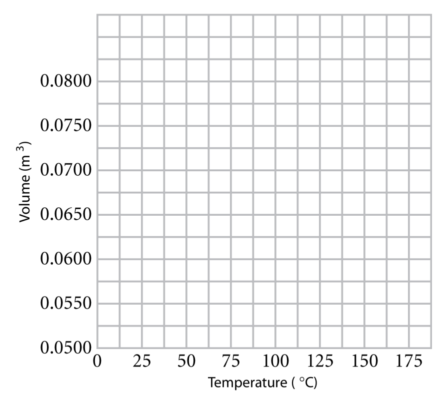
      $a + b =$ __________ $a - b =$ __________

      $a \times b =$ __________ $a \div b =$ __________
5. Calculate the area of a carpet 6.35 m long and 2.50 m wide. Express your answer with the correct number of significant figures.

   ____
6. The table below contains measurements of the temperature and volume of an air balloon as it heats up.

   In the grid at right, sketch a graph that best describes these data.

Temperature (°C)	Volume (m^3)
2	0.0502
27	0.0553
52	0.0598
77	0.0646
102	0.0704
127	0.0748
152	0.0796



Chapter 2

HOLT PHYSICS

Mixed Review

Motion in One Dimension

1. During a relay race along a straight road, the first runner on a three-person team runs d_1 with a constant velocity v_1. The runner then hands off the baton to the second runner, who runs d_2 with a constant velocity v_2. The baton is then passed to the third runner, who completes the race by traveling d_3 with a constant velocity v_3.

   a. In terms of d and v, find the time it takes for each runner to complete a segment of the race.

   Runner 1 ________ Runner 2 ________ Runner 3 ________

   b. What is the total distance of the race course?

   c. What is the total time it takes the team to complete the race?

2. The equations below include the equations for straight-line motion. For each of the following problems, indicate which equation or equations you would use to solve the problem, but do not actually perform the calculations.

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t \qquad \Delta x = \tfrac{1}{2}(v_f)\Delta t$$
$$\Delta x = v_i(\Delta t) + \tfrac{1}{2}a(\Delta t)^2 \qquad \Delta x = \tfrac{1}{2}a(\Delta t)^2$$
$$v_f = v_i + a(\Delta t) \qquad v_f = a(\Delta t)$$
$$v_f^2 = v_i^2 + 2a\Delta x \qquad v_f^2 = 2a\Delta x$$

   a. During takeoff, a plane accelerates at 4 m/s^2 and takes 40 s to reach takeoff speed. What is the velocity of the plane at takeoff?

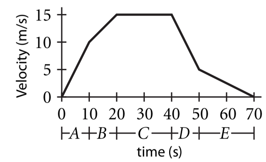
   b. A car with an initial speed of 31.4 km/h accelerates at a uniform rate of 1.2 m/s^2 for 1.3 s. What is the final speed and displacement of the car during this time?

Chapter 2

HOLT PHYSICS

Mixed Review *continued*

3. Below is the velocity-time graph of an object moving along a straight path. Use the information in the graph to fill in the table below.



For each of the lettered intervals below, indicate the motion of the object (whether it is speeding up, slowing down, or at rest), the direction of the velocity (+, –, or 0), and the direction of the acceleration (+, –, or 0).

Time interval	Motion	v	a
A			
B			
C			
D			
E			

4. A ball is thrown upward with an initial velocity of 9.8 m/s from the top of a building.

   a. Fill in the table below showing the ball's position, velocity, and acceleration at the end of each of the first 4 s of motion.

Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
1			
2			
3			
4			

   b. In which second does the ball reach the top of its flight?

   c. In which second does the ball reach the level of the roof, on the way down?

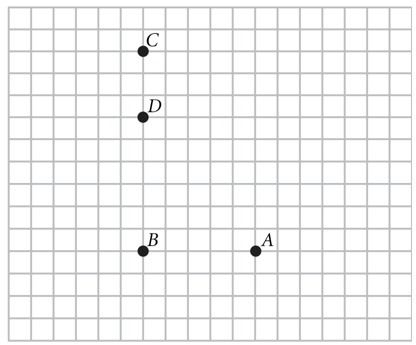
Chapter 3

HOLT PHYSICS

Mixed Review

Two-Dimensional Motion and Vectors

1. The diagram below indicates three positions to which a woman travels. She starts at position *A*, travels 3.0 km to the west to point *B*, then 6.0 km to the north to point *C*. She then backtracks, and travels 2.0 km to the south to point *D*.



   a. In the space provided, diagram the displacement vectors for each segment of the woman's trip.

   b. What is the total displacement of the woman from her initial position, *A*, to her final position, *D*?

   c. What is the total distance traveled by the woman from her initial position, *A*, to her final position, *D*?

2. Two projectiles are launched from the ground, and both reach the same vertical height. However, projectile B travels twice the horizontal distance as projectile A before hitting the ground.

   a. How large is the vertical component of the initial velocity of projectile B compared with the vertical component of the initial velocity of projectile A?

   b. How large is the horizontal component of the initial velocity of projectile B compared with the horizontal component of the initial velocity of projectile A?

   c. Suppose projectile A is launched at an angle of 45° to the horizontal. What is the ratio, v_B/v_A, of the speed of projectile B, v_B, compared with the speed of projectile A, v_A?

Chapter 3

HOLT PHYSICS

Mixed Review *continued*

3. A passenger at an airport steps onto a moving sidewalk that is 100.0 m long and is moving at a speed of 1.5 m/s. The passenger then starts walking at a speed of 1.0 m/s in the same direction as the sidewalk is moving. What is the passenger's velocity relative to the following observers?

   a. A person standing stationary alongside to the moving sidewalk.

   b. A person standing stationary *on* the moving sidewalk.

   c. A person walking alongside the sidewalk with a speed of 2.0 m/s and in a direction opposite the motion of the sidewalk.

   d. A person riding in a cart alongside the sidewalk with a speed of 5.0 m/s and in the same direction in which the sidewalk is moving.

   e. A person riding in a cart with a speed of 4.0 m/s and in a direction perpendicular to the direction in which the sidewalk is moving.

4. Use the information given in item 3 to answer the following questions:

   a. How long does it take for the passenger walking on the sidewalk to get from one end of the sidewalk to the other end?

   b. How much time does the passenger save by taking the moving sidewalk instead of walking alongside it?

Chapter 4

HOLT PHYSICS

Mixed Review

Forces and the Laws of Motion

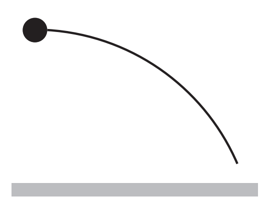
1. A crate rests on the horizontal bed of a pickup truck. For each situation described below, indicate the motion of the crate relative to the ground, the motion of the crate relative to the truck, and whether the crate will hit the front wall of the truck bed, the back wall, or neither. Disregard friction.

   a. Starting at rest, the truck *accelerates* to the right.

   b. The crate is at rest relative to the truck while the truck moves to the right with a constant velocity.

   c. The truck in item b slows down.

2. A ball with a mass of m is thrown through the air, as shown in the figure.



   a. What is the gravitational force exerted on the ball by Earth?

   b. What is the force exerted on Earth by the ball?

   c. If the surrounding air exerts a force on the ball that resists its motion, is the *total* force on the ball the same as the force calculated in part a?

   d. If the surrounding air exerts a force on the ball that resists its motion, is the *gravitational* force on the ball the same as the force calculated in part a?

3. Two blocks of masses m_1 and m_2, respectively, are placed in contact with each other on a smooth, horizontal surface. A constant horizontal force F to the right is applied to m_1. Answer the following questions in terms of F, m_1, and m_2.

   a. What is the acceleration of the two blocks?

   b. What are the horizontal forces acting on m_2?

   c. What are the horizontal forces acting on m_1?

   d. What is the magnitude of the contact force between the two blocks?

4. Assume you have the same situation as described in item 3, only this time there is a frictional force, F_k, between the blocks and the surface. Answer the following questions in terms of F, F_k, m_1, and m_2.

   a. What is the acceleration of the two blocks?

   b. What are the horizontal forces acting on m_2?

   c. What are the horizontal forces acting on m_1?

   d. What is the magnitude of the contact force between the two blocks?

Chapter 5

HOLT PHYSICS

Mixed Review

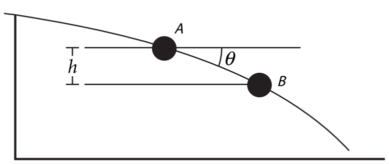
Work and Energy

1. A ball has a mass of 3 kg. What is the work done on this ball by the gravitational force exerted by Earth if the ball moves 2 m along each of the following directions?

   a. downward (along the force) ____________

   b. upward (opposite the force) ____________

2. A stone with a mass of m is thrown off a building. As the stone passes point A, it has a speed of v_A at an angle of q to the horizontal. The stone then travels a vertical distance h to point B, where it has a speed v_B.



   a. What is the work done on the stone by the gravitational force due to Earth while the stone moves from A to B?

   b. What is the change in the kinetic energy of the stone as it moves from A to B?

   c. What is the speed v_B of the stone in terms of v_A, g, and h?

   d. Does the change in the stone's speed between A and B depend on the mass of the stone?

   e. Does the change in the stone's speed between A and B depend on the angle q?

3. An empty coffee mug with a mass of 0.40 kg gets knocked off a tabletop 0.75 m above the floor onto the seat of a chair 0.45 m above the floor. Assume that the gravitational potential energy, PE_g, is measured using the floor as the zero energy level.

   a. What is the initial gravitational potential energy associated with the mug's position on the table?

   b. What is the final gravitational potential energy associated with the mug's position on the chair seat?

   c. What was the work done by the gravitational force as it fell from the table to the chair?

   d. Suppose that zero level for the energy was taken to be the ceiling of the room rather than the floor . Would the answers to items a to c be the same or different?

4. A carton of shoes with a mass of m slides with an initial speed of v_i m/s down a ramp inclined at an angle of 23° to the horizontal. The carton's initial height is h_i, and its final height is h_f, and it travels a distance of d down the ramp. There is a frictional force, F_k, between the ramp and the carton.

   a. What is the initial mechanical energy, ME_i, of the carton? (Hint: Apply the law of conservation of energy.)

   b. If mis the coefficient of friction between the ramp and the carton, what is F_k?

   c. Express the final speed, v_f, of the carton in terms of v_i, g, d, and m

Chapter 6

HOLT PHYSICS

Mixed Review

Momentum and Collisions

1. A pitcher throws a softball toward home plate. The ball may be hit, sending it back toward the pitcher, or it may be caught, bringing it to a stop in the catcher's mitt.

   a. Compare the change in momentum of the ball in these two cases.

   b. Discuss the magnitude of the impulse on the ball in these two cases.

   c. In the space below, draw a vector diagram for each case, showing the initial momentum of the ball, the impulse exerted on the ball, and the resulting final momentum of the ball.

2. a. Using Newton's third law, explain why the impulse on one object in a collision is equal in magnitude but opposite in direction to the impulse on the second object.

   b. Extend your discussion of impulse and Newton's third law to the case of a bowling ball striking a set of 10 bowling pins.

3. Starting with the conservation of total momentum, $\mathbf{p_f} = \mathbf{p_i}$, show that the final velocity for two objects in an inelastic collision is

$$\mathbf{v_f} = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v_{1,i}} + \left(\frac{m_2}{m_1 + m_2}\right)\mathbf{v_{2,i}}.$$

4. Two moving billiard balls, each with a mass of *M*, undergo an elastic collision. Immediately before the collision, ball *A* is moving east at 2 m/s and ball B is moving east at 4 m/s.

   a. In terms of *M*, what is the total momentum (magnitude and direction) immediately before the collision?

   b. The final momentum, $M(\mathbf{v_{A,f}} + \mathbf{v_{B,f}})$, must equal the initial momentum. If the final velocity of ball *A* increases to 4 m/s east because of the collision, what is the final momentum of ball *B*?

   c. For each ball, compare the final momentum of the ball to the initial momentum of the other ball. These results are typical of head-on elastic collisions. What generalization about head-on elastic collisions can you make?

Chapter 7

HOLT PHYSICS

Mixed Review

Rotational Motion and the Law of Gravity

1. Complete the following table.

	s (m)	r (m)	Δq (rad)	Δt (s)	w (rad/s)	v_t (m/s)	a_c (m/s^2)
a.	4.5		1.5	0.50			
b.		0.50	8.5		8.5		
c.	3.2	0.20			58		
d.	1250		2.0	17			
e.	3750	750				86	

2. Describe the force that maintains circular motion in the following cases.

a. A car exits a freeway and moves around a circular ramp to reach the street below.

b. The moon orbits Earth.

c. During gym class, a student hits a tether ball on a string.

3. Determine the change in gravitational force under the following changes.

a. one of the masses is doubled ______

b. both masses are doubled ______

c. the distance between masses is doubled ______

d. the distance between masses is halved ______

e. the distance between masses is tripled ______

Chapter 7

HOLT PHYSICS

Mixed Review *continued*

4. Some plans for a future space station make use of rotational force to simulate gravity. In order to be effective, the centripetal acceleration at the outer rim of the station should equal about 1 g, or 9.81 m/s^2. However, humans can withstand a difference of only 1/100 g between their head and feet before they become disoriented. Assume the average human height is 2.0 m, and calculate the minimum radius for a safe, effective station. (Hint: The ratio of the centripetal acceleration of astronaut's feet to the centripetal acceleration of the astronaut's head must be at least 99/100.)

5. As an elevator begins to descend, you feel momentarily lighter. As the elevator stops, you feel momentarily heavier. Sketch the situation, and explain the sensations using the forces in your sketch.

6. Two cars start on opposite sides of a circular track. One car has a speed of 0.015 rad/s; the other car has a speed of 0.012 rad/s. If the cars start p radians apart, calculate the time it takes for the faster car to catch up with the slower car.

Chapter 8

HOLT PHYSICS

Mixed Review

Rotational Equilibrium and Dynamics

1. a. On some doors, the doorknob is in the center of the door. What would a physicist say about the practicality of this arrangement? Why would physicists design doors with knobs farther from the hinge?

   b. How much more force would be required to open the door from the center rather than from the edge?

2. Figure skaters commonly change the shape of their body in order to achieve spins on the ice. Explain the effects on each of the following quantities when a figure skater pulls in his or her arms.

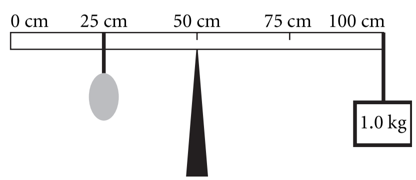
   a. moment of inertia

   b. angular momentum

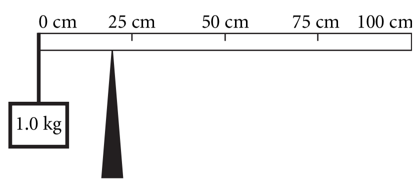
   c. angular speed

3. For the following items, assume the objects shown are in rotational equilibrium.

   a. What is the mass of the sphere to the right?



   b. What is the mass of the portion of the meter-stick to the left of the pivot? (Hint: 20% of the mass of the meterstick is on the left. How much must be on the right?)



Chapter 8 HOLT PHYSICS Mixed Review *continued*

4. A force of 25 N is applied to the end of a uniform rod that is 0.50 m long and has a mass of 0.75 kg.

   a. Find the torque, moment of inertia, and angular acceleration if the rod is allowed to pivot around its center of mass.

   b. Find the torque, moment of inertia, and angular acceleration if the rod is allowed to pivot around the end, away from the applied force.

5. A satellite in orbit around Earth is initially at a constant angular speed of 7.27×10^{-5} rad/s. The mass of the satellite is 45 kg, and it has an orbital radius of 4.23×10^{7} m.

   a. Find the moment of inertia of the satellite in orbit around Earth.

   b. Find the angular momentum of the satellite.

   c. Find the rotational kinetic energy of the satellite around Earth.

   d. Find the tangential speed of the satellite.

   e. Find the translational kinetic energy of the satellite.

6. A series of two simple machines is used to lift a 13300 N car to a height of 3.0 m. Both machines have an efficiency of 0.90 (90 percent). Machine A moves the car, and the output of machine B is the input to machine A.

   a. How much work is done on the car?

   b. How much work must be done on machine A in order to achieve the amount of work done on the car?

   c. How much work must be done on machine B in order to achieve the amount of work from machine A?

   d. What is the overall efficiency of this process?

Mixed Review Answers

Chapter 1 Mixed Review

1. a. 2.2×10^5 s
   b. 3.5×10^7 mm
   c. 4.3×10^{-4} km
   d. 2.2×10^{-5} kg
   e. 6.71×10^{11} mg
   f. 8.76×10^{-5} GW
   g. 1.753×10^{-1} ps
2. a. 3
   b. 4
   c. 10
   d. 3
   e. 2
   f. 4
3. a. 4
   b. 5
   c. 3
4. a. 1.0054; −0.9952; 5.080×10^{-3}; 5.076×10^{-3}
   b. 4.597×10^7; 3.866×10^7; 1.546×10^{14}; 11.58
5. 15.9 m^2
6. The graph should be a straight line.

Chapter 2 Mixed Review

1. a. $t_1 = d_1/v_1$; $t_2 = d_2/v_2$; $t_3 = d_3/v_3$
   b. total distance $= d_1 + d_2 + d_3$
   c. total time $= t_1 + t_2 + t_3$
2. a. $v_f = a(\Delta t)$
   b. $v_f = v_i + a(\Delta t)$; $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ or $\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2$

3.

Time interval	Type of motion	v(m/s)	a(m/s²)
A	speeding up	+	+
B	speeding up	+	+
C	constant velocity	+	0
D	slowing down	+	–
E	slowing down	+	–

4. a.

Time (s)	Position (m)	v(m/s)	a(m/s²)
1	4.9	0	−9.81
2	0	−9.8	−9.81
3	−14.7	−19.6	−9.81
4	−39.2	−29.4	−9.81

   b. 1 s
   c. 2 s

Chapter 3 Mixed Review

1. a. The diagram should indicate the relative distances and directions for each segment of the path.
   b. 5.0 km, slightly north of northwest
   c. 11.0 km
2. a. The same
   b. Twice as large
   c. 1.58
3. a. 2.5 m/s, in the direction of the sidewalk's motion
   b. 1.0 m/s, in the direction of the sidewalk's motion
   c. 4.5 m/s, in the direction of the sidewalk's motion
   d. 2.5 m/s, in the direction opposite to the sidewalk's motion
   e. 4.7 m/s, q = 32°
4. a. 4.0×10^1 seconds
   b. 6.0×10^1 seconds

Chapter 4 Mixed Review

1. a. at rest, moves to the left, hits back wall
   b. moves to the right (with velocity v), at rest, neither
   c. moves to the right, moves to the right, hits front wall
2. a. mg, down
   b. mg, up
   c. no
   d. yes
3. a. $a = \dfrac{F}{m_1 + m_2}$
   b. $m_2 a$
   c. $F - m_2 a = m_1 a$
   d. $\left(\dfrac{m_1}{m_1 + m_2}\right)F$
4. a. $a = \dfrac{F - F_k}{m_1 + m_2}$
   b. $m_2 a - F_k$
   c. $F - m_2 a - F_k = m_1 a - F_k$
   d. $\left(\dfrac{m_1}{m_1 + m_2}\right)(F - F_k)$

Chapter 5 Mixed Review

1. a. 60 J
   b. −60 J
2. a. mgh
   b. mgh
   c. $v_B = \sqrt{v_A^2 + 2gh}$
   d. no
   e. no
3. a. 2.9 J
   b. 1.8 J
   c. 1.2 J
   d. a, b: different; c: same
4. a. $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f + F_k d$
   b. $F_k = \mu mg(\cos 23°)$
   c. $v_f = \sqrt{mv_i^2 + 2g(d \sin 23° - \mu \cos 23°)}$

Chapter 6 Mixed Review

1. a. The change due to the bat is greater than the change due to the mitt.
   b. The impulse due to the bat is greater than the impulse due to the mitt.
   c. Check student diagrams. Bat: vector showing initial momentum and a larger vector in the opposite direction showing impulse of bat, result is the sum of the vectors. Mitt: vector showing initial momentum and an equal length vector showing impulse of mitt, result is the sum, which is equal to zero.
2. a. The impulses are equal, but opposite forces, occurring during the same time interval.
   b. The total force on the bowling ball is the sum of forces on pins. The force on the pins is equal but opposite of total force on ball.
3. $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$;
   $m_1 v_{1i}/(m_1 + m_2) + m_2 v_{2i}/(m_1 + m_2) = v_f$
4. a. M(6 m/s)
   b. 2 m/s
   c. objects trade momentum; if masses are equal, objects trade velocities

Chapter 7 Mixed Review

1. a. 3.0, 3.0, 9.0, 27
   b. 4.3, 1.0, 4.3, 37
   c. 16, 0.28, 11, 6.0×10^2
   d. 630, 0.11,74, 8.7
   e. 5.0, 44, 0.11, 9.9
2. a. friction
   b. gravitational force
   c. tension in string
3. a. doubled
   b. quadrupled
   c. reduced to $\frac{1}{4}$
   d. quadrupled
   e. reduced to $\frac{1}{9}$
4. 190 m
5. Student diagrams should show vectors for weight and normal force from elevator; descent should show normal force less than weight; stopping should show normal force greater than weight; "weightlessness" feeling is due to acceleration.
6. 1050 s (17.5 min)

Chapter 8 Mixed Review

1. a. If the knob is farther from the hinge, torque is increased torque for a given force.
   b. twice as much
2. a. Rotational inertia is reduced.
   b. Angular momentum remains the same.
   c. Angular speed increases.
3. a. 2.0 kg
   b. 0.67 kg
   c. 2.1×10^8 J
   d. 3.1×10^3 m/s
   e. 2.2×10^8 J
4. a. 6.2 N•m, 0.016 kg•m^2, 390 rad/s^2
   b. 12 N•m, 0.062 kg•m^2, 190 rad/s^2
5. a. 8.1×10^{16} kg•m^2
   b. 5.9×10^{12} kg•m^2/s
6. a. 4.0×10^4 J
   b. 4.4×10^4 J
   c. 4.9×10^4 J
   d. 0.81