Physics Review Notes

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The most recent version of this can be found at http://www.tomstrong.org/physics/

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These notes are meant to be a summary of important points covered in the Physics class at Mt. Lebanon High School. They are not meant to be a replacement for your own notes that you take in class, nor are they a replacement for your textbook.

This is a work in progress and will be changing and expanding over time. I have attempted to verify the correctness of the information presented here, but the final responsibility there is yours. Before relying on the information in these notes please verify it against other sources.

Chapter 1 — About Science

1.1 The Basic Science — Physics

Physics is the most basic of the living and non-living sciences. All other sciences are built on a knowledge of physics. We can understand science in general much better if we understand physics first.

1.2 Mathematics — The Language of Science

Physics equations are systems of connections following all of the rules of logic. They not only tell us what is connected to what, they tell us what we can ignore. Mathematical equations are unambiguous, they don't have the multiple meanings that often confuse the discussions of ideas expressed in common language.

1.3 The Scientific Method

The scientific method is a process that is extremely effective in gaining, organizing, and applying new knowledge. The general form of the scientific method is:

1. Recognize a problem through observation.
2. Make an educated guess (a hypothesis) about the answer.
3. Predict the consequences of the hypothesis.
4. Perform one or more experiments to test the hypothesis.
5. Analyze and interpret the results of the experiment(s)
6. Share your conclusions with others in a way that they can independently verify your results.

There are many ways of stating the scientific method, this is just one of them. Some steps above are sometimes combined, at other times one step may be divided into two or more. The important part is not the number of steps listed or their grouping but is instead the general process and the emphasis on deliberate reproduceable action.

1.4 The Scientific Attitude

In science a **fact** is just a close agreement by competent observers who make a series of observations of the same phenomenon. In other words, it is what is generally believed by the scientific community to be true.

A **hypothesis** is an educated guess that is only presumed to be factual until verified or contradicted through experiment.

When hypotheses are tested repeatedly and not contradicted they may be accepted as fact and are then known as **laws** or **principles**.

If a scientist finds evidence that contradicts a law, hypothesis, or principle then (unless the contradicting evidence turns out to be wrong) that law, hypothesis, or principle must be changed to fit the new data or abandoned if it can not be changed.

In everyday speech a theory is much like a hypothesis in that it generally indicates something that has yet to be verified. In science a **theory** is instead the result of well-tested and verified hypotheses about the reasons for certain observed behaviors. Theories are refined as new information is obtained.

Scientific facts are statements that describe what happens in the world that can be revised when new evidence is found, scientific theories are interpretations of the facts that explain the reasons for what happens.

1.5 Scientific Hypotheses Must Be Testable

When a hypothesis is created it is more important that there be a means of proving it wrong than there be a means of proving it correct. If there is no test that could disprove it then a hypothesis is not scientific.

1.6 Science, Technology, and Society

Science is a method of answering theoretical questions, technology is a method for solving practical problems. Scientists pursue problems from their own interest or to advance the general body of knowledge, technologists attempt to design, create or build something for the use or enjoyment of people.

1.7 Science, Art, and Religion

Science, art, and religion all involve the search for order and meaning in the world, and while they each go back thousands of years and overlap in several ways, all three exist for different purposes. Art works to describe the human experience and communicate emotions, science describes natural order and predicts what is possible in nature, and religion involves nature's purpose.

Accuracy vs. Precision

- **Accuracy** describes how close a measured value is to the true value of the quantity being measured

  Problems with accuracy are due to error. To avoid error:

  - Take repeated measurements to be certain that they are consistent (avoid human error)
  - Take each measurement in the same way (avoid method error)
  - Be sure to use measuring equipment in good working order (avoid instrument error)

Counting Significant Figures in a Number

Rule	Example
All counted numbers have an infinite number of significant figures	10 items, 3 measurements
All mathematical constants have an infinite number of significant figures	$1/2$, π, e
All nonzero digits are significant	42 has two significant figures; 5.236 has four
Always count zeros between nonzero digits	20.08 has four significant figures; 0.00100409 has six
Never count leading zeros	042 and 0.042 both have two significant figures
Only count trailing zeros if the number contains a decimal point	4200 and 420000 both have two significant figures; 420. has three; 420.00 has five
For numbers in scientific notation apply the above rules to the mantissa (ignore the exponent)	4.2010×10^{28} has five significant figures

Counting Significant Figures in a Calculation

Rule	Example
When adding or subtracting numbers, find the number which is known to the fewest decimal places, then round the result to that decimal place.	$21.398 + 405 - 2.9 = 423$ (3 significant figures, rounded to the ones position)
When multiplying or dividing numbers, find the number with the fewest significant figures, then round the result to that many significant figures.	$0.049623 \times 32.0/478.8 = 0.00332$ (3 significant figures)
When raising a number to some power count the number's significant figures, then round the result to that many significant figures.	$5.8^2 = 34$ (2 significant figures)
Mathematical constants do not influence the precision of any computation.	$2 \times \pi \times 4.00 = 25.1$ (3 significant figures)
In order to avoid introducing errors during multi-step calculations, keep extra significant figures for intermediate results then round properly when you reach the final result.	

- **Precision** refers to the degree of exactness with which a measurement is made and stated.
  - 1.325 m is more precise than 1.3 m
  - lack of precision is usually a result of the limitations of the measuring instrument, not human error or lack of calibration
  - You can estimate where divisions would fall between the marked divisions to increase the precision of the measurement

Chapter 2 — Linear Motion

2.1 Motion is Relative

To measure the motion of an object it muse be measured **relative** to something else, in other words some reference needs to be there to measure the motion against. You can measure the motion of a car moving down the highway, but if you measure it from another moving car you will get very different measurements than if you are a person standing beside the road.

2.2 Speed

A moving object travels a certain distance in a given time. **Speed** is a measure of how fast something is moving or how fast distance is covered. It is measured in terms of length covered per unit time so you will encounter units like meters per second or miles per hour when working with speed.

Instantaneous Speed

The speed that an object is moving at some instant is the **instantaneous speed** of that object. If you are in a car you can find the instantaneous speed by looking at the speedometer. The instantaneous speed can change at any time and my be changing continuously.

Average Speed

The average speed is a measure of the total distance covered in some amount of time. In the car from the example above the average speed is the total distance for the trip divided by the total time for the trip, the car could have been traveling faster or slower than that speed during parts of the trip, the average is only concerned with the total distance and the total time. Mathematically, you can find the average speed as

$$average\ speed = \frac{total\ distance\ covered}{time\ interval}$$

2.3 Velocity

In everyday language **velocity** is understood to mean the same as speed, but in physics there is an important distinction. Speed and velocity both describe the rate at which something is moving, but in addition to the rate velocity also includes the direction of movement. You could describe the speed of a car as 60 km per hour, but the car's velocity could be 60 km per hour to the north, or to the south, or in any other direction so long as it is specified. You can change an object's velocity without changing its speed by causing it to move in another direction but changing an object's speed will always also change the velocity.

2.4 Acceleration

The rate at which an object's velocity is changing is called its **acceleration**. It is a measure of how the velocity changes with respect to time, so

$$acceleration = \frac{change\ in\ velocity}{time\ interval}$$

Be sure to realize that it is the change in velocity, not the velocity itself that involves acceleration. A person on a bicycle riding at a constant velocity of 30 km per hour has zero acceleration regardless of the time that they ride as long as the velocity remains constant.

Acceleration in a negative direction will cause an object to slow down, this is often referred to as deceleration or negative acceleration.

2.5 Free Fall: How Fast

When something is dropped from a height it will fall toward the earth because of gravity. If there is no air resistance (something that we will usually assume) then the object will fall with a constant acceleration and be in a state known as **free fall**.

If you were to start a stopwatch at the instant the object was dropped the stopwatch would measure the **elapsed time** for the fall.

All objects in free fall will fall with the same acceleration, that is the acceleration caused by gravity and it is given the symbol g. A precise value would be $g = 9.81\ \frac{\text{m}}{\text{s}^2}$ but for most purposes we will make the math a bit easier and use $g = 10\ \frac{\text{m}}{\text{s}^2}$. (Use $g = 10\ \frac{\text{m}}{\text{s}^2}$ unless you are told otherwise.)

For an object starting at rest the relationship between the instantaneous velocity of the object (v) and the elapsed time (t) is

$$v = gt$$

This equation will only work for an object dropped from rest, if the object is thrown upward or downward then the problem has to be broken up into smaller pieces, taking advantage of the relationship between the speed of the object at the same elevation as shown on page 18 of your textbook.

2.6 Free Fall: How Far

If an object is in free fall and dropped from rest then the relationship between the distance the object has fallen (d), the elapsed time (t), and the gravitational acceleration constant (g) is

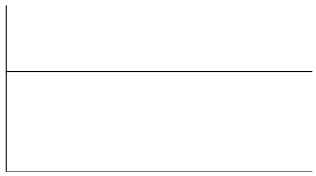
$$d = \frac{1}{2}gt^2$$

The mathematical models for free fall will also work for anything else moving in a straight line with constant acceleration, all you need to do is replace the gravitational acceleration constant (g) with the acceleration of the object you are studying (a)

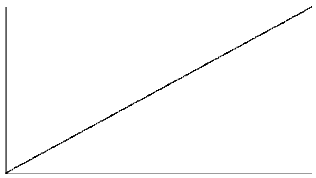
$$v = at \qquad d = \frac{1}{2}at^2$$

2.7 Graphs of Motion

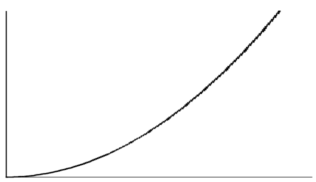
For an object starting at rest and moving with a constant acceleration a typical set of graphs of acceleration, velocity, and distance traveled would look like this when graphed with respect to time



Acceleration — a is a constant



Velocity — $v = at$



Distance — $d = \frac{1}{2}at^2$

At every point the value of the velocity graph is the slope of the distance graph, and the value of the acceleration graph is the slope of the velocity graph. Similarly, at every point the velocity graph is equal to the area under the acceleration graph to that point and the displacement graph is equal to the area under the velocity graph to that point.

2.8 Air Resistance & Falling Objects

When actual objects are dropped air resistance will cause different objects to fall in slightly different ways. For example, a brick will fall about the same way whether there is air resistance or not since it is heavy and compact but a piece of paper will tend to float downward. Wadding up the paper into a ball will cause it to fall faster by decreasing the area that has to move through the air but it will still fall slower than a brick dropped at the same time.

2.9 How Fast, How Far, How Quickly How Fast Changes

Velocity is the rate of change of how far something has traveled, acceleration is the rate of change of the velocity or the rate of change of the rate of change of the distance. This may sound confusing at first, if so back up and look at it a piece at a time as it was initially explained in the earlier parts of the chapter.

Chapter 3 — Projectile Motion

3.1 Vector & Scalar Quantities

A **vector** is something that requires both **magnitude** (size) and direction for a complete description. This could be how far a rock has fallen (20 m downward), how quickly a car is accelerating (10 meters per second per second forward), how hard something is being pushed (with 35 newtons of force to the left), or any number of other quantities.

If something can be completely described with just magnitude alone then it is known as a **scalar** quantity. Examples of scalars could be an elapsed time (22 seconds), a mass (15 kilograms) or a volume (0.30 cubic meters). Trying to add a direction to a scalar quantity (15 kilograms to the right) would not have any useful meaning in physics.

3.2 Velocity Vectors

When vectors are drawn they are customarily drawn so that their length is proportional to the magnitude of the quantity they represent. If a group of vectors are being added their sum (the **resultant**) can be found by drawing each of the vectors to scale and in the proper direction so that one vector starts where the previous one ends. If everything is drawn to scale then the resultant will just be another vector drawn from the beginning of the first vector to the end of the last one. (See pages 30 and 31 of your textbook for examples of how this works)

3.3 Components of Vectors

Just as any two (or more) vectors representing the same quantity (all velocity, all force, etc. — you can't add a force to a velocity) can be added to find a single resultant vector you can also take any single vector and break it into two pieces that are at right angles to each other. This is called **resolution** of the vector into **components**. This can be done by drawing the vector to scale and at the proper angle, finding a rectangle that will just fit around it, and then measuring the sides of the rectangle (as shown in your book on page 31) or you can do it mathematically using sine and cosine if you prefer.

3.4 Projectile Motion

Any object that is shot, thrown, dropped, or otherwise winds up moving through the air (or even above the air in some cases) is known as a **projectile**. The horizontal and vertical motion of a projectile are independent of each other, the horizontal motion is just motion with a constant velocity, vertically it is just free fall. Pages 33 and 34 of your textbook have several examples of this type of motion.

3.5 Upwardly Launched Projectiles

Because of gravity anything launched upward at an angle will follow a curved path and (usually, unless it is moving extremely fast) return to the earth. If there was no gravity then if you threw a softball it would travel forever in a straight line, because of gravity the softball will fall away from that straight line just as much as if it was dropped. (After 1 second it will be 5 m below the line, after 2 seconds it will be 20 m below it, after 3 seconds 45 m, etc.)

The components of the velocity will change in very different ways as the projectile moves. The horizontal component will not change at all, the vertical component will be the same as for another object thrown straight up. You can take advantage of this to find how long something will be in the air from one of the components, then, using the time that you just found, find the other missing pieces of the problem.

An object thrown at 45° will travel farther than at any other angle, one thrown straight up will reach a higher maximum height.

For an object launched with an initial velocity v_i at an angle of θ above the ground the distance it will travel is

$$d = \frac{v_i^2 \sin(2\theta)}{g}$$

where g is just the familiar gravitational acceleration constant.

3.6 Fast-Moving Projectiles — Satellites

If something is thrown extremely fast (faster than about 8 km per second) then it will never fall to the earth, instead the earth will curve away faster than the object will fall and it will go into orbit as a **satellite**.

Chapter 4 — Newton's First Law of Motion — Inertia

4.1 Aristotle on Motion

The Greek scientist Aristotle divided motion into natural motion (objects moving straight up or straight down, heading toward their eventual resting place) and violent motion (motion imposed by an external cause moving an object away from its resting place). According to Aristotle it is the nature of any object to come to rest and the object will do this on its own, it does not need any external influence for this to happen.

4.2 Copernicus and the Moving Earth

Copernicus reasoned that the simplest way to explain astronomical observations was to assume that the the Earth and other planets moved around the sun instead of the common belief that the Earth was at the center off the universe. This was contrary to Aristotle's teachings which were widely accepted at the time and caused Copernicus to delay the publication of his findings until almost the end of his life.

4.3 Galileo on Motion

On of Galileo's contributions to physics was the idea that a **force** is not necessary to keep an object moving.

A force is any push or pull on an object. **Friction** is the force that occurs when two objects rub against each other and the small surface irregularities create a force that opposes the moving object(s). Galileo argued that only when friction is present is a force necessary to keep an object moving. The **inertia** of an object is a measure of its tendency to keep moving as it is currently moving.

Galileo studied how objects moved rather than why. He showed that experiments instead of logic were the best test of ideas.

4.4 Newton's Law of Inertia

Newton's first law, often called the **inertia law** states *every object continues in a state of rest, or of motion in a straight line at constant speed, unless it is compelled to change that state by forces exerted upon it.*

Another way of saying this is that objects will continue to do whatever they are currently doing unless something causes them to change. No force is required to maintain motion, a force is needed only to change it.

4.5 Mass — A Measure of Inertia

Mass is the amount of matter in an object or more specifically a measure of the inertia of an object that will resist changes in motion.

The **weight** (F_g) of an object is the gravitational force upon it, it can be found with the equation

$$weight = mass \times gravitational\ acceleration$$

or

$$F_g = mg$$

4.6 Net Force

The **net force** acting on an object is the sum of all of the forces acting on it. They may cancel each other out either totally or partially or they may combine to produce a force greater than any of the individual forces on the object.

4.7 Equilibrium — When Net Force Equals Zero

When the net force on an object is zero the object is in a state of **equilibrium**. This could be because all of the forces are canceling each other out or it could be because there is actually no force on the object, either way the object will not accelerate.

4.8 Vector Addition of Forces

Forces are added as vectors just as displacement, velocity, and acceleration are. The same methods of vector addition will work that you have used previously. Your textbook has several examples of vector addition of forces on pages 52–54.

4.9 The Moving Earth Again

All measurements of motion are made relative to the observer. If you are in a moving car you can look at objects in the car that appear stationary to you but in fact the are moving with you. The same thing happens with respect to the Earth. Since the Earth moves everyone on it will move with it and we will not notice that motion, everything moves on the Earth as if the Earth was not moving.

Chapter 5 — Newton's Second Law of Motion — Force and Acceleration

5.1 Force Causes Acceleration

If the net force acting on an object is no zero the object will be accelerated, the acceleration of the object will be directly proportional to the net force on the object — if the net force is doubled the acceleration will also be doubled.

5.2 Mass Resists Acceleration

The larger the mass of an object the smaller the acceleration that will be produced by a constant net force. The exact relationship is that the acceleration is inversely proportional to the mass, another way of saying that is that the acceleration is proportional to one divided by the mass of the object.

5.3 Newton's Second Law

Newton's second law states that *the acceleration produced by a net force on an object is directly proportional to the magnitude of the net force, is in the same direction as the net force, and is inversely proportional to the mass of the object.*

Mathematically, if F is the net force, Newton's second law can be expressed as

$$F = ma \qquad \text{or} \qquad a = \frac{F}{m}$$

5.4 Friction

Friction is a force that acts between any two objects in contact with each other to oppose their relative motion. The force of friction depends on the surfaces in contact (rougher surfaces produce more friction) as well as the force pressing the surfaces together (the more force pressing the surfaces together the more friction there will be). When frictional forces are present they must be accounted for along with other applied forces to calculate the net force on objects.

5.5 Applying Force — Pressure

When two objects exert force on each other the **pressure** on the surface in contact can be found as

$$pressure = \frac{force}{area\ of\ contact}$$

The pressure can be increased by applying more force over the same area or by applying the same force over a smaller area.

5.6 Free Fall Explained

The weight of an object is the force that causes an object to fall, and the weight is the product of the mass and the gravitational acceleration constant. Since $F = ma$ the acceleration is equal to the force divided by the mass, this yields:

$$a = \frac{F}{m} = \frac{mg}{m} = \frac{\cancel{m}g}{\cancel{m}} = g$$

In the absence of air resistance or other frictional forces the acceleration of a falling object is equal to g, the gravitational acceleration constant.

5.7 Falling and Air Resistance

At low speeds air resistance (R) is very small and can usually be ignored for most objects, as the speed increases so does the air resistance. The heavier and smaller the object (such as a steel ball) the less air resistance will affect it, larger and lighter objects (such as a feather or a piece of paper) will be affected more because of the combination of a larger area and a smaller mass.

Chapter 6 — Newton's Third Law of Motion — Action and Reaction

6.1 Forces and Interactions

A force is any push or pull on an object, but it also involves a second object to produce the force. This mutual action, or **interaction** actually causes there to be a pair of forces — if a hammer applies a force to a nail the nail will also apply a force back on the hammer.

6.2 Newton's Third Law

Newton's third law states *whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first object.*

One force is called the **action force**, the other force is called the **reaction force.** Neither force can exist without the other and they are equal in strength and opposite in direction. Newton's third law is often expressed as "for every action there is an equal and opposite reaction".

6.3 Identifying Action and Reaction

When analyzing forces the action force may be apparent but the reaction force is often harder to see until you know what to look for. Whatever the action force is, the reaction force always involves the same two objects. If the action force is the Earth pulling down on you with your weight the reaction force would be you pulling up on the Earth with the same force. The two objects do not have to be in contact to interact.

6.4 Action and Reaction on Different Masses

Action and reaction forces always act on a pair of objects. When a rifle is fired the action force is the rifle acting on the bullet, the reaction force is the bullet pushing back on the rifle. Since the bullet has a small mass and the rifle has a large mass the bullet will have a larger acceleration.

6.5 Do Action and Reaction Forces Cancel?

Since action and reaction forces each act upon different objects they can never cancel each other. In order to cancel each other out forces must all act upon the same object.

6.6 The Horse-Cart Problem

See the explanation and illustrations on pages 80–82 of your textbook for an excellent explanation of how action-reaction pairs work and how to handle multiple force pairs in one problem.

6.7 Action Equals Reaction

In order to apply a force to an object that object must be able to apply an equal and opposite force to you. The example that they use is trying to apply a 200 N punch to a piece of paper — when you apply even a small force to the paper it will easily accelerate out of your way from the applied force. Since it can't stay in one place to apply a reaction force you cannot apply the larger action force.

Chapter 7 — Momentum

7.1 Momentum

Momentum is a vector quantity described by the product of an object's mass times its velocity:

$$p = mv$$

Momentum is also known as inertia in motion. An object must be moving to have momentum, objects at rest have none.

7.2 Impulse Changes Momentum

A force acting on an object for some amount of time produces an **impusle** which will change the object's momentum. The impulse is defined as the product of the force and the time the force acts, so

$$impulse = Ft$$

Since impulse is also the change in momentum this then becomes

$$\Delta p = Ft$$

The same impulse can be delivered by a small force acting over a long time or a large force acting for a short time. If an object will be brought to a stop increasing the time during which the force acts will cause the force to be reduced, this is the principle behind padding, air bags, car bumpers, and many other safety devices.

7.3 Bouncing

When an object is brought to a stop the change in momentum is equal to the momentum that the object originally had. If the object bounces then the change in momentum will be twice the original momentum of the object, so an object that bounces will deliver twice the impulse of one that comes to a stop.

7.4 Conservation of Momentum

When two objects interact the momentum lost by one object will be gained by the other. No momentum will be created or destroyed, it will only be moved around between the objects. When momentum (or any other quantity, such as mass or energy) acts like this we say that it is **conserved**. This leads to one of the principla laws of mechanics, the **law of conservation of momentum** which states *In the absence of an external force, the momentum of a system remains unchanged.* The external force being referred to in this case is one coming from an object not in the system.

7.5 Collisions

When two objects collide we will consider two cases, one where the two objects bounce off of each other (known as an **elastic collision**) and another where they stick together and travel as a single combined object (known as an **inelastic collision**). In either case since momentum is conserved the momenta of the two objects before and after the collision will follow the relationship

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

7.6 Momentum Vectors

Since momentum is a vector quantity it can act in any number of directions, not just in a straight line. The diagrams in your textbook on page 98 help to illustrate what will happen in a collision when the two objects colliding do not both move in the same line.

Chapter 8 — Energy

8.1 Work

When a force acts on an object over some distance (moving the object parallel to the direction of the force) the force is said to do **work** (W) on the object, the amount of work done is the product of the force times the distance over which the force was applied:

$$W = Fd$$

Work is measured in newtons times meters which are given the name **joules** (J).

8.2 Power

Power (P) is defined to be the rate at which work is done, or the work done divided by the time interval during which the work was done:

$$P = \frac{W}{t}$$

Power is measured in joules divided by seconds which are given the name **watts** (W). One watt of power is what is required to convert one joule of energy every second.

8.3 Mechanical Energy

Energy is the accumulation of work done in an object. If work is done in making an object move faster it has the ability to do work on something else when it slows down (**kinetic energy**), if the work is done in raising it to some height then it has the ability to do work as it is lowered back to its original position (**potential energy**). Just like work energy is measured in joules.

8.4 Potential Energy

Potential energy (PE) is the energy stored in an object by lifting it to a higher position, the force required to lift the object it its weight (mg) and the distance over which that force acts is the height (h), so the product of them is the potential energy:

$$PE = mgh$$

Potential energy is a relative measurement, you choose the height that will be your zero level when you set up your frame of reference.

8.5 Kinetic Energy

Kinetic energy (KE) is the energy an object has because it is moving and it is equal to the work done in accelerating the object to the speed at which it is traveling. The more mass or speed the object has the larger the kinetic energy:

$$KE = \frac{1}{2}mv^2$$

8.6 Conservation of Energy

Energy can be transferred from one object to another by one of them doing work on the other, it can also be transferred from one kind of energy to another within an object. This leads to the **law of conservation of energy** which states that *energy cannot be created or destroyed. It can be transformed from one form into another, but the total amount of energy never changes.* If anything exerts a force on an object in a system then the source of that force must also be included in the system for the law of conservation of energy to hold, otherwise energy could be moved into or out of the system.

For a single object moving without friction or other interactions the law of conservation of energy states that the sum of the kinetic and potential energies for the object will always be the same, so

$$PE_i + KE_i = PE_f + KE_f$$

or, by expanding those terms

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

8.7 Machines

A **machine** is a device that is used to multiply forces or change the direction of forces. All machines are subject to the law of conservation of energy, no machine can produce more energy than is fed into it. In the ideal case no work will be lost to friction so

$$W_{out} = W_{in}$$

And since $W = Fd$ this can become

$$F_{out}d_{out} = F_{in}d_{in}$$

The amount that the machine multiplies the input force to get the output force is called the **mechanical advantage** (MA) of the machine:

$$MA = \frac{F_{out}}{F_{in}}$$

In the absence of friction the **theoretical mechanical advantage** (TMA) can also be expressed as the ratio of the input distance to the output distance

$$TMA = \frac{d_{in}}{d_{out}}$$

Some common machines are shown in your textbook on pages 112-115.

8.8 Efficiency

The **efficiency** of a machine is the ratio of work output to work input, or

$$eff = \frac{W_{out}}{W_{in}}$$

It can also be expressed as the ratio between the actual mechanical advantage (MA) and the theoretical mechanical advantage (TMA):

$$eff = \frac{MA}{TMA}$$

8.9 Energy for Life

Living cells can also be regarded as machines, they take energy from either fuel or the sun and convert it to other forms.

Chapter 9 — Circular Motion

9.1 Rotation and Revolution

If an object rotates around an **axis** that passes through the object (an internal axis) then the motion is described as **rotation**, if the axis does not pass through the object then the motion is described as **revolution**. As an example, every day the Earth rotates around its axis, once a year it revolves around the sun.

9.2 Rotational Speed

The speed of a rotating object can be measured different ways, first the **rotational speed** (also known as **angular speed**, ω) measures how many rotations occur in a unit of time, for a rigid body (something that will not change shape as you are studying it) every point on the object will have the same angular speed. The other type of speed is the **tangential speed** (v), that measures how fast a single point on the object is moving when measured along a straight line that is tangent to the motion of that point. The farther an object is from the axis the larger the tangential speed will be, a point on the axis will have a tangential speed of zero. The two speeds are related through the equation

$$tangential\ speed = radial\ distance \times angular\ speed$$

or

$$v = r\omega$$

9.3 Centripetal Force

If an object is moving with no net force acting on it then Newton's first law says it will move in a straight line. To keep an object moving in a circular path requires a constant force toward the center known as a **centripetal force** (a center-seeking force). For an object of mass m moving in a circle of radius r with tangential speed v and angular speed ω the centripetal force, F_c, is found by

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

The corresponding centripetal acceleration (a_c) is

$$a_c = \frac{v^2}{r} = r\omega^2$$

9.4 Centripetal and Centrifugal Forces

It it important to realize that the force causing circular motion is a centripetal force, not at force pulling away from the axis of rotation (a **centrifugal force** or center-fleeing force). There is no centrifugal force causing circular motion.

9.5 Centrifugal Force in a Rotating Reference

If you are in some object that is moving in a circular path you might feel like you are experiencing a centrifugal force but in actuality there is no such force, what you are experiencing is the reaction force to the object pushing you toward the center of the circle. No outward force exists on an object as a result of the object's motion in a circle.

9.6 Simulated Gravity

If a space station were constructed in the shape of a large wheel (as shown in the diagrams on page 131 of your textbook) and the wheel were rotated about the axis then anyone on the inside of the wheel would feel that they were being pulled downward as if by gravity. This is caused by the centripetal force that keeps them moving in a circle, their reaction force will seem to pull them "down" toward the outside rim of the station.

The magnitude of the force acceleration or force experienced for a given rotational speed is proportional to the radial distance to the "floor" so a larger space station would have a larger simulated gravity than a smaller one. A smaller space station would also give a greater difference in apparent gravitational force from a person's head to their feet than a larger one would.

Chapter 10 — Center of Gravity

10.1 Center of Gravity

When studying physical objects it eventually becomes necessary to consider their size and shape. Up until now we have been looking at objects as if they could be represented by a single point. If an object has definite size and shape then it should be possible to find a single point to represent the object, that point is the **center of gravity** of the object.

If an object is thrown through the air or slid across a frictionless table while it spins then it is the center of gravity of the object that will trace out a smooth curve. All of the other parts of the object will rotate around the center of gravity when it spins.

10.2 Center of Mass

Center of gravity is often also called **center of mass**, in most cases there is no difference although if an object is large enough for the gravitational attraction to vary from one side to the other (such as a very tall building) then there may be a very small difference between the two points. In this class we will often use the two terms interchangeably.

10.3 Locating the Center of Gravity

The center of gravity of a uniform object is at the geometric center of the object, for a more complicated object it can be found by suspending the object from different points, tracing a line straight down from the point of suspension, repeating this, and then seeing where the lines cross — this point of intersection is the center of gravity of the object.

The center of gravity of an object may be located where no actual material of the object exists, for example the center of gravity of a donut is in the middle of the hole.

10.4 Toppling

Every object that rests on a surface will contact that surface in one or more points, if you were to imagine a rubber band stretched around those points the area enclosed by that rubber band is the object's supporting base. As long as the center of gravity is above (or hanging below) that supporting base the object will remain in place. If the center of gravity moves to no longer be over (or under) the supporting base then the object will fall over.

10.5 Stability

An object with no net force acting on it can be either easy or difficult to tip over, depending on the size of the supporting base of the object and how close the center of gravity is to the edge of the supporting base.

If an object is placed so that its center of gravity is directly over the edge of its supporting base then any small displacement will start to lower the center of gravity and the object will tip over, this condition is known as **unstable equilibrium**.

If the object is placed so that its center of gravity is not on the edge of its supporting base then a small displacement will start to raise the center of gravity and the object will return to the initial position when released, this is known ad **stable equilibrium**.

A third condition also exists where the distance from the center of gravity to the outside of an object is the same for any (or most) orientations of the object, in that case a small displacement will neither raise nor lower the center of gravity and the object will remain in the new, displaced, position, this is known as **neutral equilibrium**.

An object or system will tend to behave in a way that will lower its center of gravity — tall objects will fall over, balls will roll down hill, water will run downward through a pipe, and heavier objects will settle to the bottom while lighter ones rise to the top in a mixture.

10.6 Center of Gravity of People

A person's center of gravity is about halfway between their front and back and a few centimeters below their navel if they are standing up straight, if they change their body position then the center of gravity will change as well. As you move around you will generally shift your body to keep your center of mass somewhere over your supporting base.

Chapter 11 — Rotational Mechanics

11.1 Torque

A force exerted some distance from the axis of rotation of an object is known as a **torque**. If a force is located on a line that passes through the axis of rotation (or the center of mass of the object if there is no fixed axis) it will cause the object to move in a straight line instead, no force passing through the axis of rotation can cause any torque.

The torque produced by a force is equal to **lever arm** (the distance from the axis of rotation to the point where the force is acting) times the component of the force that is perpendicular to the lever arm, mathematically that would be

$$torque = lever\ arm \times perpendicular\ force$$

The same torque can be produced by a large force with a short lever arm or a small force with a long lever arm.

11.2 Balanced Torques

If the sum of the torques in a clockwise direction is equal to the sum of the torques in the counterclockwise direction (a net torque of zero) then the object the torques are acting on is said to be in **rotational equilibrium**. Two people on a non-moving seesaw or a balanced scale would be examples of objects in rotational equilibrium.

See page 153 of your textbook for an example of how to use balanced torques to find unknown forces or distances.

11.3 Torque and Center of Gravity

Any force passing through the center of gravity of an object will tend to move the object instead of rotating it, if the force instead passes some distance from the center of gravity then the object will experience a torque that will tend to cause the object to rotate as well as move.

11.4 Rotational Inertia

An object's resistance to rotation is known as its **rotational inertia**, also known as **moment of inertia**. Just as an object with larger mass is more difficult to start or stop moving than one with a small mass, an object with a large moment of inertia is more difficult to start or stop rotating than one with a small moment of inertia.

The farther the mass of an object is from its axis of rotation the larger the object's moment of inertia, an object with the same mass closer to the axis of rotation will have a smaller moment of inertia. For an object of mass m with all of its mass a distance r from the axis of rotation the moment of inertia I will be

$$I = mr^2$$

Moments of inertia of more complicated objects are listed in your textbook on page 157.

Since objects with large moments of inertia are more resistant to changing their rotational speed than those with smaller moments of inertia if two or more objects are rolled down an inclined plane the first one to reach the bottom will be the one with the smallest moment of inertia, a solid cylinder will beat a ring and a sphere will beat both the cylinder and the ring.

11.5 Rotational Inertia and Gymnastics

A human body can be considered to have three principal axes as pictured on page 159 of your textbook. They are the longitudinal, transverse, and median axes. A body's moment of inertia is least around the longitudinal axis and about equal around the other two.

11.6 Angular Momentum

Just as an object moving in a straight line has momentum equal to the product of its mass times its velocity an object that is rotating has **angular momentum** equal to the product of its moment of inertia times its angular speed. Unlike linear momentum which is always the same for any mass and velocity the angular momentum of an object will vary depending on the arrangement of the object's mass in addition to the mass and angular velocity.

11.7 Conservation of Angular Momentum

Angular momentum is conserved, a rotating object will always have the same amount unless the angular momentum is transferred to some other object. This, in combination with the dependence on the arrangement of mass will allow an object to change its rotational speed if the arrangement of mass changes, a notable example of this is standing on a rotating platform while holding weights, if you start with your arms outstretched you will gain speed when you pull your arms in despite your angular momentum remaining constant.

Chapter 12 — Universal Gravitation

12.1 The Falling Apple

Newton observed that falling objects (such as the infamous apple) were just one example of gravitational attraction, there is a gravitational attraction between every pair of objects no matter their size or distance.

12.2 The Falling Moon

In the absence of some external force the moon should continue to move in a straight line. Since it does not move in a straight line but instead orbits the earth there must be some force acting on it. This turns out to be the gravitational attraction between the earth and the moon. Every second the moon falls toward the earth a very small amount relative to where it would be if it kept traveling in a straight line, that distance that the moon falls is about 1.4 mm every second.

12.3 The Falling Earth

Just as the moon falls around the earth, the earth and all of the other planets fall around the sun. If they ever stopped moving they would fall straight in toward the sun.

12.4 Newton's Law of Universal Gravitation

Newton discovered that gravitational attraction is a universal force. Every bit of matter in the universe attracts every other bit of matter, no matter how small they are or how far apart they may be. The exact force of gravitational attraction, F_g, between any two objects of masses m_1 and m_2 separated by a distance r is

$$F_g = G\frac{m_1 m_2}{r^2}$$

where G, the constant of universal gravitation, is found experimentally to be

$$G = 6.673 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

12.5 Gravity and Distance: The inverse Square Law

Because the denominator of the equation above includes the square of the distance gravity is said to follow the inverse square law. As the distance increases between two objects their force of attraction will decrease by the square of the amount the distance increased, for example doubling the distance will produce 1/4 of the force, tripling it will produce 1/9 of the force, etc. There is a figure on page 176 in your textbook that helps to explain this phenomenon.

12.6 Universal Gravitation

As discussed above, every particle of matter in the universe attracts every other particle. This has led to the discovery of the planet Neptune because of the observed effects of its gravitational attraction on the planet Uranus, and the same thing happened to lead to the discovery of the former planet Pluto.

Chapter 13 — Gravitational Interactions

13.1 Gravitational Fields

Gravity is a field force (a force that will act between two objects not in contact) and the strength of the gravitational field at any point is the acceleration that would be caused on an object at that point. At the surface of the earth the gravitational field strength is the familiar gravitational acceleration constant, $g = 9.8\ \frac{\text{m}}{\text{s}^2}$.

To find the gravitational field strength g at any point a distance r from the center of mass of a planet of mass m we use a modification of the universal gravitation equation

$$g = G\frac{m}{r^2}$$

where $G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

13.2 Gravitational Field Inside a Planet

The equation above for the gravitational field strength assumes that all of the mass of the planet is on one side of you, if you burrow down into the planet then some of the planet's mass will be on the other side of you, that will partially cancel the force from the rest of the mass and reduce the effect. At the center of the planet the gravitational attraction from one side would be exactly balanced by the attraction from the other side and you would feel no net gravitational force.

13.3 Weight and Weightlessness

If you are in an enclosure that is being accelerated you will feel a change in your apparent weight, think about being in an elevator that is changing speed, if it's starting to move upward you will feel heavier than normal, if it's starting to move downward you will feel lighter than normal. If somehow the supporting cables were to break then you and the elevator car would both fall at the same rate and you would feel no weight relative to the elevator.

13.4 Ocean Tides

Since the gravitational pull depends on the distance between the objects there is a difference in the gravitational pull from the moon to the near and far sides of the earth. For a solid object this would not be noticeable but for a liquid like the water in the oceans it means that the water is pulled more on the side closer to the moon and less on the side away from the moon. This gives a tidal bulge on the side toward the moon (pulled there by the moon) and another one on the side away from the moon (where there is a smaller pull from the moon and the oceans move away). There are also similar tidal bulges produced by the sun but because the sun is much farther away they are less than half the size of those produced by the moon.

13.5 Tides and the Earth and Atmosphere

The earth itself is not a completely rigid object, it's made up of a slightly flexible crust over a less solid inside. Because of this the earth's surface itself is affected by the tidal forces. It's not a noticeable effect since everything on the surface moves with the surface but it is measurable and has been found to be as much as 25 cm.

13.6 Black Holes

Stars are large accumulations of mostly gas that are constantly being affected by two forces — the gravitational force pulling the gas in toward the center and the nuclear fusion in the core that pushes the gas back out. If the nuclear fusion weakens over time the gravitational force will pull the gas inward, and if there is enough gas there to pull inward then eventually the star will collapse inward onto itself and form what is known as a black hole. (Smaller stars will form what is known as a back dwarf, effectively a cinder that is all that's left of a burned-out star.)

The amount of mass present in the black hole is exactly the same as the mass in the star that collapsed so anything that was orbiting the star will orbit the black hole instead. The difference comes with the size of the black hole — since it is much smaller than the star it is now possible to get much closer to the center than was previously possible. Getting closer will greatly increase the gravitational force in that area and eventually the pull of gravity is so large that even light can't get away from the center.

Chapter 14 — Satellite Motion

14.1 Earth Satellites

A satellite is any object that falls around the earth instead of falling down into it. Since a projectile would drop about 5 m every second and the earth's curvature is such that you have to travel about 8000 m for the surface to curve away 5 m, if you are traveling at 8000 m per second near the surface of the earth then you will never hit the earth — you will always fall around it instead. The result of this would be that instead of hitting the earth the object would instead be in a very low orbit.

As a practical matter if you are moving close to the surface of the earth then the friction of the atmosphere will quickly slow you down so most satellites stay about 150 km above the earth to avoid the atmosphere.

14.2 Circular Orbits

If an object is in a circular orbit then it is always moving at the same speed and always at the same distance from the earth. There is no component of the gravitational force in the direction of the satellite's motion so there can be no acceleration on the satellite.

The **period** of the satellite is the time that it takes for one complete orbit, if the satellite is orbiting close to the earth (within a few hundred kilometers of the surface) then the period is about 90 minutes.

A special kind of circular orbit is what is known as a **geosynchronous** or **geostationary** orbit, that is an orbit over the equator with a period of 24 hours and will cause the satellite to always be above the same point on the earth. This is used for communications satellites and others that always need to be in the same position relative to the earth so that they can be found by transmitters and receivers on the ground.

14.3 Elliptical Orbits

If a satellite close to the earth has a speed higher than 8 km per second then it will overshoot a circular orbit and move farther from the earth, trading off kinetic energy for potential energy as it slows down while increasing altitude. The path traced out will be an ellipse with the earth at one focus. (A circle is a special case of an ellipse with only one focus.)

The speed of the satellite will be smallest at the highest point (called the **apogee**) and largest at the lowest point (called the **perigee**).

14.4 Energy Conservation and Satellite Motion

The total energy (kinetic plus potential) of any satellite is constant. For an elliptical orbit one kind of energy is constantly being traded for the other, for a circular orbit both kinetic and potential energy remain constant, but in either case their sum is always the same.

14.5 Escape Speed

The **escape speed** of an object is the speed that something must travel to completely leave the earth (or another planet or star). For an object on the surface of the earth that speed is 11.2 km per second. Anything launched slower than that speed will eventually return to earth, anything faster will not. If the object starts from farther away then the escape speed will be reduced, similarly if the planet has a smaller mass then the escape will also be reduced. There is a chart in your textbook on page 208 with the escape speeds from several bodies in the solar system.

Variables and Notation

SI Prefixes

Prefix	Mult.	Abb.	Prefix	Mult.	Abb.
yocto-	10^{-24}	y	yotta-	10^{24}	Y
zepto-	10^{-21}	z	zetta-	10^{21}	Z
atto-	10^{-18}	a	exa-	10^{18}	E
femto-	10^{-15}	f	peta-	10^{15}	P
pico-	10^{-12}	p	tera-	10^{12}	T
nano-	10^{-9}	n	giga-	10^{9}	G
micro-	10^{-6}	μ	mega-	10^{6}	M
milli-	10^{-3}	m	kilo-	10^{3}	k
centi-	10^{-2}	c	hecto-	10^{2}	h
deci-	10^{-1}	d	deka-	10^{1}	da

Notation

Notation	Description
$\vec{x}$	Vector
x	Scalar, or the magnitude of $\vec{x}$
$\lvert\vec{x}\rvert$	The absolute value or magnitude of $\vec{x}$
Δx	Change in x
$\sum x$	Sum of all x
Πx	Product of all x
x_i	Initial value of x
x_f	Final value of x
$\hat{x}$	Unit vector in the direction of x
$A \longrightarrow B$	A implies B
$A \propto B$	A is proportional to B
$A \gg B$	A is much larger than B

Units

Symbol	Unit	Quantity	Composition
kg	kilogram	Mass	SI base unit
m	meter	Length	SI base unit
s	second	Time	SI base unit
A	ampere	Electric current	SI base unit
cd	candela	Luminous intensity	SI base unit
K	kelvin	Temperature	SI base unit
mol	mole	Amount	SI base unit
Ω	ohm	Resistance	$\frac{V}{A}$ or $\frac{m^2 \cdot kg}{s^3 \cdot A^2}$
C	coulomb	Charge	$A \cdot s$
F	farad	Capacitance	$\frac{C}{V}$ or $\frac{s^4 \cdot A^2}{m^2 \cdot kg}$
H	henry	Inductance	$\frac{V \cdot s}{A}$ or $\frac{m^2 \cdot kg}{A^2 \cdot s^2}$
Hz	hertz	Frequency	s^{-1}
J	joule	Energy	$N \cdot m$ or $\frac{kg \cdot m^2}{s^2}$
N	newton	Force	$\frac{kg \cdot m}{s^2}$
rad	radian	Angle	$\frac{m}{m}$ or 1
T	tesla	Magnetic field	$\frac{N}{A \cdot m}$
V	volt	Electric potential	$\frac{J}{C}$ or $\frac{m^2 \cdot kg}{s^3 \cdot A}$
W	watt	Power	$\frac{J}{s}$ or $\frac{kg \cdot m^2}{s^3}$
Wb	weber	Magnetic flux	$V \cdot s$ or $\frac{kg \cdot m}{s^2 \cdot A}$

Greek Alphabet

Name	Maj.	Min.	Name	Maj.	Min.
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π or ϖ
Epsilon	E	ϵ or ε	Rho	P	ρ or ϱ
Zeta	Z	ζ	Sigma	Σ	σ or ς
Eta	H	η	Tau	T	τ
Theta	Θ	θ or ϑ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ or φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Variables

Variable	Description	Units
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ	Angular position	rad
θ_c	Critical angle	^{o} (degrees)
θ_i	Incident angle	^{o} (degrees)
θ_r	Refracted angle	^{o} (degrees)
θ'	Reflected angle	^{o} (degrees)
$\Delta\theta$	Angular displacement	rad
τ	Torque	$\text{N} \cdot \text{m}$
ω	Angular speed	$\frac{\text{rad}}{\text{s}}$
μ	Coefficient of friction	(unitless)
μ_k	Coefficient of kinetic friction	(unitless)
μ_s	Coefficient of static friction	(unitless)
$\vec{a}$	Acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_c$	Centripetal acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_g$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_t$	Tangential acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_x$	Acceleration in the x direction	$\frac{\text{m}}{\text{s}^2}$
$\vec{a}_y$	Acceleration in the y direction	$\frac{\text{m}}{\text{s}^2}$
A	Area	m^2
B	Magnetic field strength	T
C	Capacitance	F
$\vec{d}$	Displacement	m
$d \sin\theta$	lever arm	m
$\vec{d}_x$ or Δx	Displacement in the x direction	m
$\vec{d}_y$ or Δy	Displacement in the y direction	m
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
f	Focal length	m
$\vec{F}$	Force	N
$\vec{F}_c$	Centripetal force	N
$\vec{F}_{electric}$	Electrical force	N
$\vec{F}_g$	Gravitational force	N
$\vec{F}_k$	Kinetic frictional force	N
$\vec{F}_{magnetic}$	Magnetic force	N
$\vec{F}_n$	Normal force	N
$\vec{F}_s$	Static frictional force	N
$\vec{F}\Delta t$	Impulse	$\text{N} \cdot \text{s}$ or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$
$\vec{g}$	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$

Variable	Description	Units
h	Object height	m
h'	Image height	m
I	Current	A
I	Moment of inertia	$\text{kg} \cdot \text{m}^2$
KE or K	Kinetic energy	J
KE_{rot}	Rotational kinetic energy	J
L	Angular Momentum	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$
L	Self-inductance	H
m	Mass	kg
M	Magnification	(unitless)
M	Mutual inductance	H
MA	Mechanical Advantage	(unitless)
ME	Mechanical Energy	J
n	Index of refraction	(unitless)
p	Object distance	m
$\vec{p}$	Momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
P	Power	W
PE or U	Potential Energy	J
$PE_{elastic}$	Elastic potential energy	J
$PE_{electric}$	Electrical potential energy	J
PE_g	Gravitational potential energy	J
q	Image distance	m
q or Q	Charge	C
Q	Heat, Entropy	J
R	Radius of curvature	m
R	Resistance	Ω
s	Arc length	m
t	Time	s
Δt	Time interval	s
$\vec{v}$	Velocity	$\frac{\text{m}}{\text{s}}$
v_t	Tangential speed	$\frac{\text{m}}{\text{s}}$
$\vec{v}_x$	Velocity in the x direction	$\frac{\text{m}}{\text{s}}$
$\vec{v}_y$	Velocity in the y direction	$\frac{\text{m}}{\text{s}}$
V	Electric potential	V
ΔV	Electric potential difference	V
V	Volume	m^3
W	Work	J
x or y	Position	m

Constants

Symbol	Name	Established Value	Value Used
ϵ_0	Permittivity of a vacuum	$8.854\ 187\ 817 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}$	$8.85 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}$
ϕ	Golden ratio	1.618 033 988 749 894 848 20	
π	Archimedes' constant	3.141 592 653 589 793 238 46	
g, a_g	Gravitational acceleration constant	$9.79171\ \frac{\mathrm{m}}{\mathrm{s}^2}$ (varies by location)	$9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$
c	Speed of light in a vacuum	$2.997\ 924\ 58 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$ (exact)	$3.00 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$
e	Natural logarithmic base	2.718 281 828 459 045 235 36	
e^-	Elementary charge	$1.602\ 177\ 33 \times 10^{19}$ C	1.60×10^{19} C
G	Gravitational constant	$6.672\ 59 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg}^2}$	$6.67 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg}^2}$
k_C	Coulomb's constant	$8.987\ 551\ 788 \times 10^{9}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2}$	$8.99 \times 10^{9}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2}$
N_A	Avogadro's constant	$6.022\ 141\ 5 \times 10^{23}\ \mathrm{mol}^{-1}$	

Astronomical Data

Symbol	Object	Mean Radius	Mass	Mean Orbit Radius	Orbital Period
☽	Moon	1.74×10^6 m	7.36×10^{22} kg	3.84×10^8 m	2.36×10^6 s
☉	Sun	6.96×10^8 m	1.99×10^{30} kg	—	—
☿	Mercury	2.43×10^6 m	3.18×10^{23} kg	5.79×10^{10} m	7.60×10^6 s
♀	Venus	6.06×10^6 m	4.88×10^{24} kg	1.08×10^{11} m	1.94×10^7 s
♁	Earth	6.37×10^6 m	5.98×10^{24} kg	1.496×10^{11} m	3.156×10^7 s
♂	Mars	3.37×10^6 m	6.42×10^{23} kg	2.28×10^{11} m	5.94×10^7 s
	Ceres[1]	4.71×10^5 m	9.5×10^{20} kg	4.14×10^{11} m	1.45×10^8 s
♃	Jupiter	6.99×10^7 m	1.90×10^{27} kg	7.78×10^{11} m	3.74×10^8 s
♄	Saturn	5.85×10^7 m	5.68×10^{26} kg	1.43×10^{12} m	9.35×10^8 s
♅	Uranus	2.33×10^7 m	8.68×10^{25} kg	2.87×10^{12} m	2.64×10^9 s
♆	Neptune	2.21×10^7 m	1.03×10^{26} kg	4.50×10^{12} m	5.22×10^9 s
♇	Pluto[1]	1.15×10^6 m	1.31×10^{22} kg	5.91×10^{12} m	7.82×10^9 s
	Eris[2][1]	2.4×10^6 m	1.5×10^{22} kg	1.01×10^{13} m	1.75×10^{10} s

[1] Ceres, Pluto, and Eris are classified as "Dwarf Planets" by the IAU
[2] Eris was formerly known as 2003 UB_{313}

Mathematics Review for Physics

This is a summary of the most important parts of mathematics as we will use them in a physics class. There are numerous parts that are completely omitted, others are greatly abridged. Do not assume that this is a complete coverage of any of these topics.

Algebra

Fundamental properties of algebra

$a + b = b + a$	Commutative law for addition
$(a + b) + c = a + (b + c)$	Associative law for addition
$a + 0 = 0 + a = a$	Identity law for addition
$a + (-a) = (-a) + a = 0$	Inverse law for addition
$ab = ba$	Commutative law for multiplication
$(ab)c = a(bc)$	Associative law for multiplication
$(a)(1) = (1)(a) = a$	Identity law for multiplication
$a\frac{1}{a} = \frac{1}{a}a = 1$	Inverse law for multiplication
$a(b + c) = ab + ac$	Distributive law

Exponents

$$(ab)^n = a^n b^n \qquad (a/b)^n = a^n/b^n$$
$$a^n a^m = a^{n+m} \qquad 0^n = 0$$
$$a^n/a^m = a^{n-m} \qquad a^0 = 1$$
$$(a^n)^m = a^{(mn)} \qquad 0^0 = 1 \text{ (by definition)}$$

Logarithms

$$x = a^y \longrightarrow y = \log_a x$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^n) = n\log_a x$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a} = (\log_b x)(\log_a b)$$

Binomial Expansions

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!}a^i b^{n-i}$$

Quadratic formula

For equations of the form $ax^2 + bx + c = 0$ the solutions are:

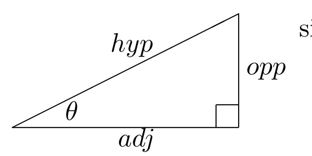
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Geometry

Shape	Area	Volume
Triangle	$A = \frac{1}{2}bh$	—
Rectangle	$A = lw$	—
Circle	$A = \pi r^2$	—
Rectangular prism	$A = 2(lw + lh + hw)$	$V = lwh$
Sphere	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Cylinder	$A = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Cone	$A = \pi r\sqrt{r^2 + h^2} + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$

Trigonometry

In physics only a small subset of what is covered in a trigonometry class is likely to be used, in particular sine, cosine, and tangent are useful, as are their inverse functions. As a reminder, the relationships between those functions and the sides of a right triangle are summarized as follows:



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

The inverse functions are only defined over a limited range. The $\tan^{-1} x$ function will yield a value in the range $-90° < \theta < 90°$, $\sin^{-1} x$ will be in $-90° \le \theta \le 90°$, and $\cos^{-1} x$ will yield one in $0° \le \theta \le 180°$. Care must be taken to ensure that the result given by a calculator is in the correct quadrant, if it is not then an appropriate correction must be made.

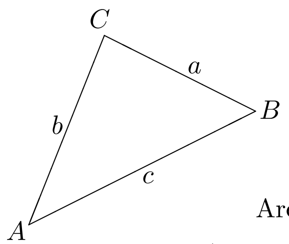
Degrees	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

$$\sin^2\theta + \cos^2\theta = 1 \qquad 2\sin\theta\cos\theta = \sin(2\theta)$$

Trigonometric functions in terms of each other

$\sin\theta =$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$
$\cos\theta =$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$
$\tan\theta =$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$
$\sec\theta =$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$
$\cot\theta =$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$
$\sin\theta =$	$\frac{1}{\csc\theta}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\frac{1}{\sqrt{\csc^2\theta-1}}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\csc\theta$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\sqrt{\csc^2\theta-1}$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Law of sines, law of cosines, area of a triangle



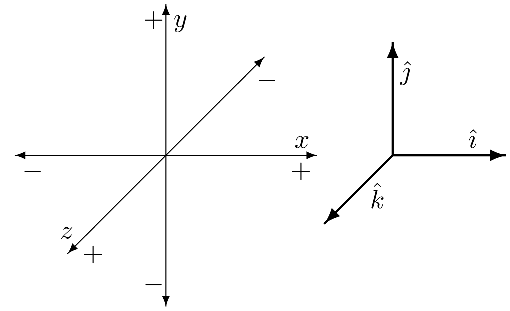
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\text{let } s = \tfrac{1}{2}(a+b+c)$$
$$\text{Area}= \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area}= \tfrac{1}{2}bc\sin A = \tfrac{1}{2}ac\sin B = \tfrac{1}{2}ab\sin C$$

Vectors

A **vector** is a quantity with both magnitude and direction, such as displacement or velocity. Your textbook indicates a vector in bold-face type as **V** and in class we have been using $\vec{V}$. Both notations are equivalent.

A **scalar** is a quantity with only magnitude. This can either be a quantity that is directionless such as time or mass, or it can be the magnitude of a vector quantity such as speed or distance traveled. Your textbook indicates a scalar in italic type as V, in class we have not done anything to distinguish a scalar quantity. The magnitude of $\vec{V}$ is written as V or $|\vec{V}|$.

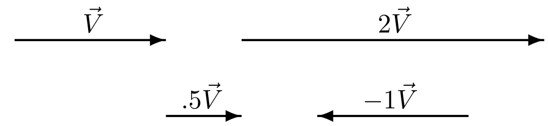
A **unit vector** is a vector with magnitude 1 (a dimensionless constant) pointing in some significant direction. A unit vector pointing in the direction of the vector $\vec{V}$ is indicated as $\hat{V}$ and would commonly be called V-hat. Any vector can be normalized into a unit vector by dividing it by its magnitude, giving $\hat{V} = \frac{\vec{V}}{V}$. Three special unit vectors, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are introduced with chapter 3. They point in the directions of the positive x, y, and z axes, respectively (as shown below).



Vectors can be added to other vectors of the same dimension (i.e. a velocity vector can be added to another velocity vector, but not to a force vector). The sum of all vectors to be added is called the **resultant** and is equivalent to all of the vectors combined.

Multiplying Vectors

Any vector can be multiplied by any scalar, this has the effect of changing the magnitude of the vector but not its direction (with the exception that multiplying a vector by a negative scalar will reverse the direction of the vector). As an example, multiplying a vector $\vec{V}$ by several scalars would give:



In addition to scalar multiplication there are also two ways to multiply vectors by other vectors. They will not be directly used in class but being familiar with them may help to understand how some physics equations are derived. The first, the **dot product** of vectors $\vec{V}_1$ and $\vec{V}_2$, represented as $\vec{V}_1 \cdot \vec{V}_2$ measures the tendency of the two vectors to point in the same direction. If the angle between the two vectors is θ the dot product yields a scalar value as

$$\vec{V}_1 \cdot \vec{V}_2 = V_1 V_2 \cos\theta$$

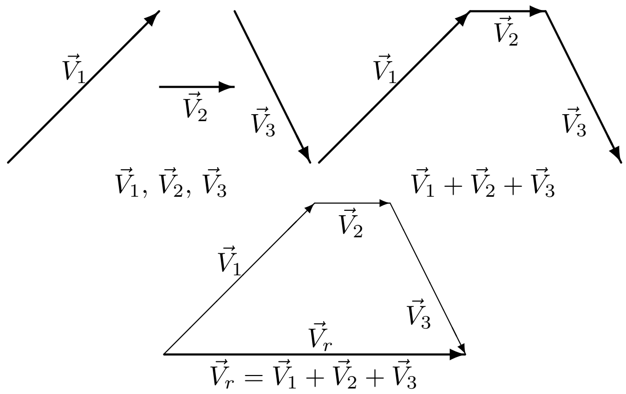
The second method of multiplying two vectors, the **cross product**, (represented as $\vec{V}_1 \times \vec{V}_2$) measures the tendency of vectors to be perpendicular to each other. It yields a third vector perpendicular to the two original vectors with magnitude

$$|\vec{V}_1 \times \vec{V}_2| = V_1 V_2 \sin\theta$$

The direction of the cross product is perpendicular to the two vectors being crossed and is found with the right-hand rule — point the fingers of your right hand in the direction of the first vector, curl them toward the second vector, and the cross product will be in the direction of your thumb.

Adding Vectors Graphically

The sum of any number of vectors can be found by drawing them head-to-tail to scale and in proper orientation then drawing the resultant vector from the tail of the first vector to the point of the last one. If the vectors were drawn accurately then the magnitude and direction of the resultant can be measured with a ruler and protractor. In the example below the vectors $\vec{V}_1$, $\vec{V}_2$, and $\vec{V}_3$ are added to yield $\vec{V}_r$

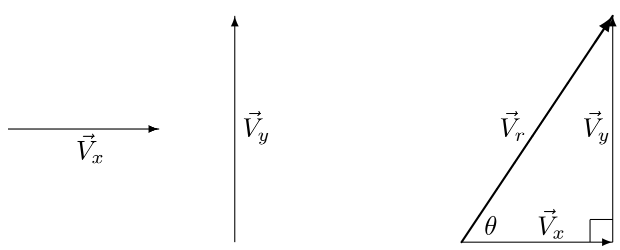


Adding Parallel Vectors

Any number of parallel vectors can be directly added by adding their magnitudes if one direction is chosen as positive and vectors in the opposite direction are assigned a negative magnitude for the purposes of adding them. The sum of the magnitudes will be the magnitude of the resultant vector in the positive direction, if the sum is negative then the resultant will point in the negative direction.

Adding Perpendicular Vectors

Perpendicular vectors can be added by drawing them as a right triangle and then finding the magnitude and direction of the hypotenuse (the resultant) through trigonometry and the Pythagorean theorem. If $\vec{V}_r = \vec{V}_x + \vec{V}_y$ and $\vec{V}_x \perp \vec{V}_y$ then it works as follows:



Since the two vectors to be added and the resultant form a right triangle with the resultant as the hypotenuse the Pythagorean theorem applies giving

$$V_r = |\vec{V}_r| = \sqrt{V_x^2 + V_y^2}$$

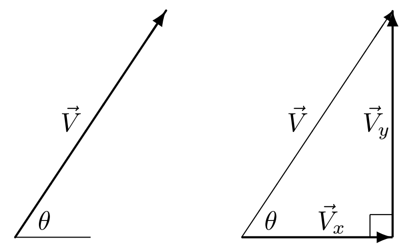
The angle θ can be found by taking the inverse tangent of the ratio between the magnitudes of the vertical and horizontal vectors, thus

$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

As was mentioned above, care must be taken to ensure that the angle given by the calculator is in the appropriate quadrant for the problem, this can be checked by looking at the diagram drawn to solve the problem and verifying that the answer points in the direction expected, if not then make an appropriate correction.

Resolving a Vector Into Components

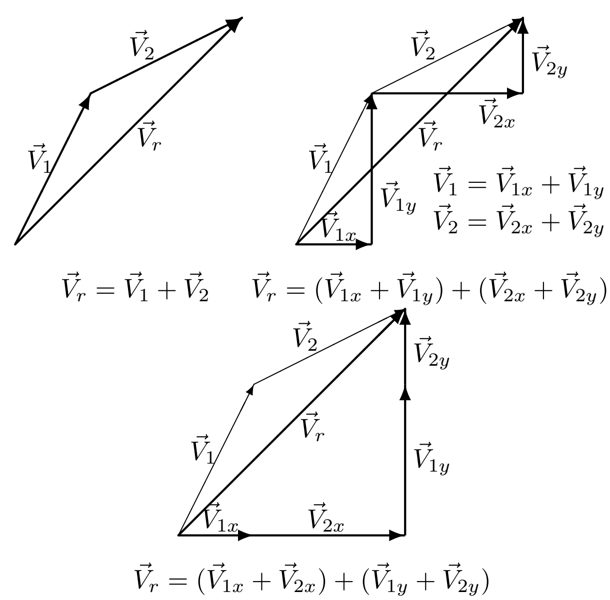
Just two perpendicular vectors can be added to find a single resultant, any single vector $\vec{V}$ can be resolved into two perpendicular **component vectors** $\vec{V}_x$ and $\vec{V}_y$ so that $\vec{V} = \vec{V}_x + \vec{V}_y$.



As the vector and its components can be drawn as a right triangle the ratios of the sides can be found with trigonometry. Since $\sin\theta = \frac{V_y}{V}$ and $\cos\theta = \frac{V_x}{V}$ it follows that $V_x = V\cos\theta$ and $V_y = V\sin\theta$ or in a vector form, $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$. (This is actually an application of the dot product, $\vec{V}_x = (\vec{V}\cdot\hat{\imath})\hat{\imath}$ and $\vec{V}_y = (\vec{V}\cdot\hat{\jmath})\hat{\jmath}$, but it is not necessary to know that for this class)

Adding Any Two Vectors Algebraically

Only vectors with the same direction can be directly added, so if vectors pointing in multiple directions must be added they must first be broken down into their components, then the components are added and resolved into a single resultant vector — if in two dimensions $\vec{V}_r = \vec{V}_1 + \vec{V}_2$ then



Once the sums of the component vectors in each direction have been found the resultant can be found from them just as an other perpendicular vectors may be added. Since from the last figure $\vec{V}_r = (\vec{V}_{1x} + \vec{V}_{2x}) + (\vec{V}_{1y} + \vec{V}_{2y})$ and it was previously established that $\vec{V}_x = V\cos\theta\hat{\imath}$ and $\vec{V}_y = V\sin\theta\hat{\jmath}$ it follows that

$$\vec{V}_r = (V_1\cos\theta_1 + V_2\cos\theta_2)\,\hat{\imath} + (V_1\sin\theta_1 + V_2\sin\theta_2)\,\hat{\jmath}$$

and

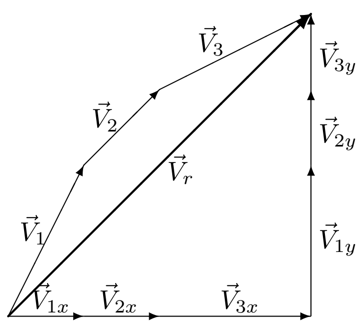
$$\begin{aligned} V_r &= \sqrt{(V_{1x} + V_{2x})^2 + (V_{1y} + V_{2y})^2} \\ &= \sqrt{(V_1\cos\theta_1 + V_2\cos\theta_2)^2 + (V_1\sin\theta_1 + V_2\sin\theta_2)^2} \end{aligned}$$

with the direction of the resultant vector $\vec{V}_r$, θ_r, being found with

$$\theta_r = \tan^{-1}\frac{V_{1y} + V_{2y}}{V_{1x} + V_{2x}} = \tan^{-1}\frac{V_1\sin\theta_1 + V_2\sin\theta_2}{V_1\cos\theta_1 + V_2\cos\theta_2}$$

Adding Any Number of Vectors Algebraically

For a total of n vectors $\vec{V}_i$ being added with magnitudes V_i and directions θ_i the magnitude and direction are:



$$\vec{V}_r = \left(\sum_{i=1}^{n} V_i\cos\theta_i\right)\hat{\imath} + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)\hat{\jmath}$$

$$V_r = \sqrt{\left(\sum_{i=1}^{n} V_i\cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} V_i\sin\theta_i\right)^2}$$

$$\theta_r = \tan^{-1}\frac{\sum_{i=1}^{n} V_i\sin\theta_i}{\sum_{i=1}^{n} V_i\cos\theta_i}$$

(The figure shows only three vectors but this method will work with any number of them so long as proper care is taken to ensure that all angles are measured the same way and that the resultant direction is in the proper quadrant.)

Calculus

Although this course is based on algebra and not calculus, it is sometimes useful to know some of the properties of derivatives and integrals. If you have not yet learned calculus it is safe to skip this section.

Derivatives

The **derivative** of a function $f(t)$ with respect to t is a function equal to the slope of a graph of $f(t)$ vs. t at every point, assuming that slope exists. There are several ways to indicate that derivative, including:

$$\frac{\mathrm{d}}{\mathrm{d}t}f(t)$$

$$f'(t)$$

$$\dot{f}(t)$$

The first one from that list is unambiguous as to the independent variable, the others assume that there is only one variable, or in the case of the third one that the derivative is taken with respect to time. Some common derivatives are:

$$\frac{\mathrm{d}}{\mathrm{d}t}t^n = nt^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin t = \cos t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\cos t = -\sin t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}e^t = e^t$$

If the function is a compound function then there are a few useful rules to find its derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}cf(t) = c\frac{\mathrm{d}}{\mathrm{d}t}f(t) \qquad c \text{ constant for all } t$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[f(t) + g(t)] = \frac{\mathrm{d}}{\mathrm{d}t}f(t) + \frac{\mathrm{d}}{\mathrm{d}t}g(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}f(u) = \frac{\mathrm{d}}{\mathrm{d}t}u\frac{\mathrm{d}}{\mathrm{d}u}f(u)$$

Integrals

The **integral**, or **antiderivative** of a function $f(t)$ with respect to t is a function equal to the the area under a graph of $f(t)$ vs. t at every point plus a constant, assuming that $f(t)$ is continuous. Integrals can be either indefinite or definite. An indefinite integral is indicated as:

$$\int f(t)\,\mathrm{d}t$$

while a definite integral would be indicated as

$$\int_a^b f(t)\,\mathrm{d}t$$

to show that it is evaluated from $t = a$ to $t = b$, as in:

$$\int_a^b f(t)\,\mathrm{d}t = \int f(t)\,\mathrm{d}t\Big|_{t=a}^{t=b} = \int f(t)\,\mathrm{d}t\Big|_{t=b} - \int f(t)\,\mathrm{d}t\Big|_{t=a}$$

Some common integrals are:

$$\int t^n\,\mathrm{d}t = \frac{t^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int t^{-1}\,\mathrm{d}t = \ln t + C$$
$$\int \sin t\,\mathrm{d}t = -\cos t + C$$
$$\int \cos t\,\mathrm{d}t = \sin t + C$$
$$\int e^t\,\mathrm{d}t = e^t + C$$

There are rules to reduce integrals of some compound functions to simpler forms (there is no general rule to reduce the integral of the product of two functions):

$$\int cf(t)\,\mathrm{d}t = c\int f(t)\,\mathrm{d}t \qquad c \text{ constant for all } t$$
$$\int [f(t) + g(t)]\,\mathrm{d}t = \int f(t)\,\mathrm{d}t + \int g(t)\,\mathrm{d}t$$

Series Expansions

Some functions are difficult to work with in their normal forms, but once converted to their series expansion can be manipulated easily. Some common series expansions are:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i+1}}{(2i+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{i=0}^{\infty}(-1)^i\frac{x^{2i}}{(2i)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{i=0}^{\infty}\frac{x^i}{(i)!}$$

Physics Using Calculus

In our treatment of mechanics the majority of the equations that have been covered in the class have been either approximations or special cases where the acceleration or force applied have been held constant. This is required because the class is based on algebra and to do otherwise would require the use of calculus.

None of what is presented here is a required part of the class, it is here to show how to handle cases outside the usual approximations and simplifications. Feel free to skip this section, none of it will appear as a required part of any assignment or test in this class.

Notation

Because the letter 'd' is used as a part of the notation of calculus the variable 'r' is often used to represent the position vector of the object being studied. (Other authors use 's' for the spatial position or generalize 'x' to two or more dimensions.) In a vector form that would become $\vec{r}$ to give both the magnitude and direction of the position. It is assumed that the position, velocity, acceleration, etc. can be expressed as a function of time as $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ but the dependency on time is usually implied and not shown explicitly.

Translational Motion

Many situations will require an acceleration that is not constant. Everything learned about translational motion so far used the simplification that the acceleration remained constant, with calculus we can let the acceleration be any function of time.

Since velocity is the slope of a graph of position vs. time at any point, and acceleration is the slope of velocity vs. time these can be expressed mathematically as derivatives:

$$\vec{v} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}$$

$$\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}$$

The reverse of this relationship is that the displacement of an object is equal to the area under a velocity vs. time graph and velocity is equal to the area under an acceleration vs. time graph. Mathematically this can be expressed as integrals:

$$\vec{v} = \int \vec{a}\,\mathrm{d}t$$

$$\vec{r} = \int \vec{v}\,\mathrm{d}t = \iint \vec{a}\,\mathrm{d}t^2$$

Using these relationships we can derive the main equations of motion that were introduced by starting with the integral of a constant velocity $\vec{a}(t) = \vec{a}$ as

$$\int_{t_i}^{t_f} \vec{a}\,\mathrm{d}t = \vec{a}\Delta t + C$$

The constant of integration, C, can be shown to be the initial velocity, $\vec{v}_i$, so the entire expression for the final velocity becomes

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

or

$$\vec{v}_f(t) = \vec{v}_i + \vec{a}t$$

Similarly, integrating that expression with respect to time gives an expression for position:

$$\int_{t_i}^{t_f} (\vec{v}_i + \vec{a}t)\,\mathrm{d}t = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2 + C$$

Once again the constant of integration, C, can be shown to be the initial position, $\vec{r}_i$, yielding an expression for position vs. time

$$\vec{r}_f(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

While calculus can be used to derive the algebraic forms of the equations the reverse is not true. Algebra can handle a subset of physics and only at the expense of needing to remember separate equations for various special cases. With calculus the few definitions shown above will suffice to predict any linear motion.

Work and Power

The work done on an object is the integral of dot product of the force applied with the path the object takes, integrated as a contour integral over the path.

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The power developed is the time rate of change of the work done, or the derivative of the work with respect to time.

$$P = \frac{\mathrm{d}}{\mathrm{d}t}W$$

Momentum

Newton's second law of motion changes from the familiar $\vec{F} = m\vec{a}$ to the statement that the force applied to an object is the time derivative of the object's momentum.

$$\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{p}$$

Similarly, the impulse delivered to an object is the integral of the force applied in the direction parallel to the motion with respect to time.

$$\Delta\vec{p} = \int \vec{F}\,\mathrm{d}t$$

Rotational Motion

The equations for rotational motion are very similar to those for translational motion with appropriate variable substitutions. First, the angular velocity is the time derivative of the angular position.

$$\vec{\omega} = \frac{\mathrm{d}\vec{\theta}}{\mathrm{d}t}$$

The angular acceleration is the time derivative of the angular velocity or the second time derivative of the angular position.

$$\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\theta}}{\mathrm{d}t^2}$$

The angular velocity is also the integral of the angular acceleration with respect to time.

$$\vec{\omega} = \int \vec{\alpha}\,\mathrm{d}t$$

The angular position is the integral of the angular velocity with respect to time or the second integral of the angular acceleration with respect to time.

$$\vec{\theta} = \int \vec{\omega}\,\mathrm{d}t = \iint \vec{\alpha}\,\mathrm{d}t^2$$

The torque exerted on an object is the cross product of the radius vector from the axis of rotation to the point of action with the force applied.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The moment of inertia of any object is the integral of the square of the distance from the axis of rotation to each element of the mass over the entire mass of the object.

$$I = \int r^2\,\mathrm{d}m$$

The torque applied to an object is the time derivative of the object's angular momentum.

$$\vec{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

The integral of the torque with respect to time is the angular momentum of the object.

$$\vec{L} = \int \vec{\tau}\,\mathrm{d}t$$

Data Analysis

Comparative Measures

Percent Error

When experiments are conducted where there is a calculated result (or other known value) against which the observed values will be compared the percent error between the observed and calculated values can be found with

$$\textit{percent error} = \left| \frac{\textit{observed value} - \textit{accepted value}}{\textit{accepted value}} \right| \times 100\%$$

Percent Difference

When experiments involve a comparison between two experimentally determined values where neither is regarded as correct then rather than the percent error the percent difference can be calculated. Instead of dividing by the accepted value the difference is divided by the mean of the values being compared. For any two values, a and b, the percent difference is

$$\textit{percent difference} = \left| \frac{a-b}{\frac{1}{2}(a+b)} \right| \times 100\%$$

Dimensional Analysis and Units on Constants

Consider a mass vibrating on the end of a spring. The equation relating the mass to the frequency on graph might look something like

$$y = \frac{3.658}{x^2} - 1.62$$

This isn't what we're looking for, a first pass would be to change the x and y variables into f and m for frequency and mass, this would then give

$$m = \frac{3.658}{f^2} - 1.62$$

This is progress but there's still a long way to go. Mass is measured in kg and frequency is measured in s^{-1}, since these variables represent the entire measurement (including the units) and not just the numeric portion they do not need to be labeled, but the constants in the equation do need to have the correct units applied.

To find the units it helps to first re-write the equation using just the units, letting some variable such as u (or u_1, u_2, u_3, etc.) stand for the unknown units. The example equation would then become

$$\mathrm{kg} = \frac{u_1}{(\mathrm{s}^{-1})^2} + u_2$$

Since any quantities being added or subtracted must have the same units the equation can be split into two equations

$$\mathrm{kg} = \frac{u_1}{(\mathrm{s}^{-1})^2} \quad \text{and} \quad \mathrm{kg} = u_2$$

The next step is to solve for u_1 and u_2, in this case $u_2 = \mathrm{kg}$ is immediately obvious, u_1 will take a bit more effort. A first simplification yields

$$\mathrm{kg} = \frac{u_1}{\mathrm{s}^{-2}}$$

then multiplying both sides by s^{-2} gives

$$(\mathrm{s}^{-2})(\mathrm{kg}) = \left(\frac{u_1}{\mathrm{s}^{-2}}\right)(\mathrm{s}^{-2})$$

which finally simplifies and solves to

$$u_1 = \mathrm{kg} \cdot \mathrm{s}^{-2}$$

Inserting the units into the original equation finally yields

$$m = \frac{3.658\ \mathrm{kg} \cdot \mathrm{s}^{-2}}{f^2} - 1.62\ \mathrm{kg}$$

which is the final equation with all of the units in place. Checking by choosing a frequency and carrying out the calculations in the equation will show that it is dimensionally consistent and yields a result in the units for mass (kg) as expected.

Midterm Exam Description

Chapter 1 — About Science

- Scientific method

Chapter 2 — Linear Motion

- Speed
- Velocity
- Acceleration
- Free fall
- Graphs of motion
- Air resistance

Chapter 3 — Projectile Motion

- Vector & scalar quantities
- Velocity vectors
- Vector components
- Projectile motion
- Satellites

Chapter 4 — Newton's 1st Law

- Aristotle, Copernicus, Galileo
- Inertia
- Net Force & equilibrium
- Adding vectors

Chapter 5 — Newton's 2nd Law

- Forces cause accelerations
- Masses resist acceleration
- Friction
- Pressure
- Free fall & air resistance

Chapter 6 — Newton's 3rd Law

- Forces & interactions
- Action & reaction on different masses
- Horse-Cart Problem
- Action = Reaction

Chapter 7 — Momentum

- Impulse changes momentum
- Bouncing
- Conservation of momentum
- Collisions
- Momentum vectors

Chapter 8 — Energy

- Work
- Power
- Mechanical energy
- Potential & kinetic energy
- Conservation of energy
- Machines
- Efficiency

Chapter 9 — Circular Motion

- Rotation and revolution
- Rotational speed
- Centripetal force
- Centrifugal force in a rotating reference
- Simulated gravity

Chapter 10 — Center of Gravity

- Center of mass
- Locating center of gravity
- Toppling
- Stability

Chapter 11 — Rotational Mechanics

- Torque
- Balanced torques
- Torque and center of gravity
- Rotational inertia
- Angular momentum
- Conservation of angular momentum

Chapters 12–14 will not appear on the semester exam.

Semester Exam Equation Sheet

This is a slightly reduced copy of the equation sheet that you will receive with the semester exam.

Physics equations

*Use **9.8 m/s^2** or **10 m/s/s** for the acceleration due to gravity on earth

Horizontal:

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$a_x = \frac{\Delta v_x}{\Delta t}$$

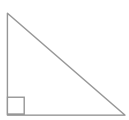
Vertical:

$$\Delta v_y = g\Delta t$$

$$d_y = \frac{1}{2} g\Delta t^2$$

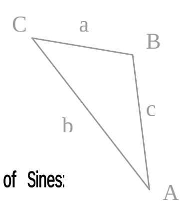
$$c^2 = a^2 + b^2$$

SOHCAHTOA



Law of Cosines:

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$



Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$w = mg$$

$$\Sigma F = ma$$

$$P_{ressure} = \frac{F}{A}$$

$$\Delta p = Ft = \Delta(mv)$$

$$(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f$$

$$(m_1v_1)_i + (m_2v_2)_i = (mv)_f$$

$$(mv)_i = (m_1v_1)_f + (m_2v_2)_f$$

$$W = Fd\cos\theta = \Delta KE$$

$$P = \frac{W}{\Delta t}$$

$$AMA = \frac{F_{out}}{F_{in}}$$

$$IMA = \frac{d_{in}}{d_{out}}$$

$$Efficiency = \frac{W_{out}}{W_{in}} \times 100 = \frac{AMA}{IMA}$$

$$PE = mgh$$

$$KE = \tfrac{1}{2}mv^2$$

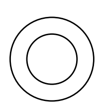
$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r} = ma_c$$

$$\tau = F_\perp \times leverarm$$

$$I = mr^2$$



$$L = I \times \omega = mvr$$

$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$v = f\lambda$$

$$f' = f_o\left(\frac{v \pm v_o}{v \pm v_s}\right)$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_1}{n_2}$$

$$F = k\frac{q_1q_2}{d^2}$$

$$I = \frac{q}{t}$$

$$V = \frac{PE}{q}$$

$$V = IR$$

$$C = \frac{q}{V}$$

$$P = \frac{W}{t} = \frac{qV}{t} = IV$$

Note that some equations may be slightly different than the ones we have used in class, the sheet that you receive will have these forms of the equations, it will be up to you to know how to use them or to know other forms that you are able to use.